

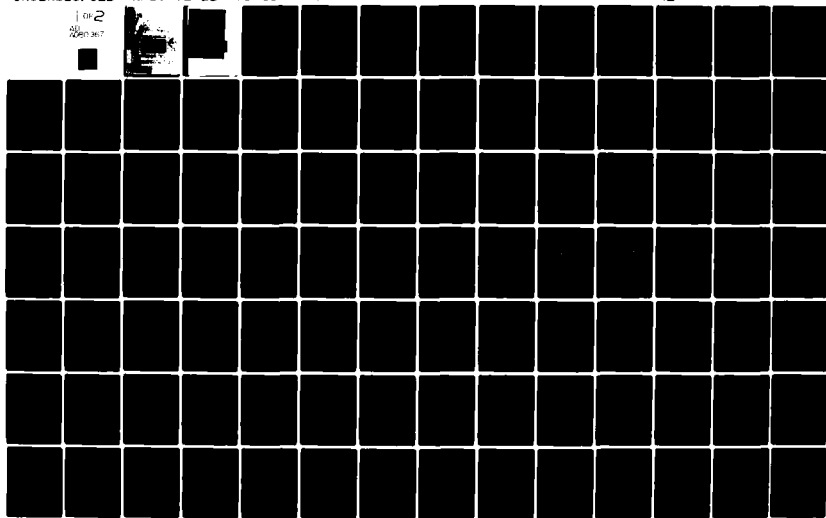
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PREDICTED MICROWAVE ELECTRO-MAGNETIC  
BACKSCATTERING RETURNS FOR SIMPLE  
REFLECTIVE TARGETS.

9 MASTER'S THESIS,

14 AFIT/GE/EE/79D-11 Frank DiBartolomeo, Jr.  
2nd Lt. USAF

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PREDICTIVE MICROWAVE ELECTROMAGNETIC  
BACKSCATTERING RETURNS FOR SIMPLE  
REFLECTIVE TARGETS

THESIS

Presented to the Faculty of the School of Engineering  
of the Air Force Institute of Technology

Air University

in Partial Fulfillment of the  
Requirements for the Degree of  
Master of Science

by

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Graduate Electrical Engineering

December 1979

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## Preface

This thesis was stimulated by research in Tactical Target Identification performed by the United States Air Force Systems Command's Rome Air Development Center (RADC/OCTM), Griffiss AFB, New York. Engineers there are investigating the possibility of identifying aircraft from two dimensional cross range versus slant range image plots of the discrete electromagnetic scatterers on the aircraft. To identify the aircraft, it is thought of as a collection of simple geometrical objects whose image plots are known at different aspect angles. For a known aircraft, the image plots of the individual geometrical objects are "pieced" together at their individual aspect angles with their spatial position observed, with respect to some reference point on the aircraft. Thus, what is produced is a composite image plot of the discrete point scatterers on the aircraft. The assumption made above is that secondary multiple scattering is negligible compared to primary scattering.

The goals of this thesis are to obtain a practical method of obtaining the backscattered signal for any known incident signal for the perfectly-conducting sphere and perfectly-conducting, infinitely-thin circular disk and also to investigate the sensitivity of the backscattered signal to changing aspect angle.

I acknowledge with thanks the advice and encouragement received from my thesis advisor, August Golden, Jr., Capt., USAF and to my

thesis readers, Joseph W. Carl, Maj., USAF, and Professor Raymond Potter at the Air Force Institute of Technology. Also, my thanks and appreciation go to my sponsor, Mr. Richard Wood of the Surveillance Division, Rome Air Development Center and to Drs. D. L. Moffat and D. B. Hodge of the Ohio State University. Dr. Hodge's computer simulation for the scattering from the circular disk was extremely helpful. Also, Capt. Golden's many helpful suggestions were invaluable to the completion of this research.

Finally, I would like to thank my wife, Kathy, whose patience and understanding were of immeasurable value throughout my AFIT program.

FRANK DIBARTOLOMEO, JR.

## Contents

Preface . . . . .	ii
List of Figures . . . . .	v
Abstract . . . . .	vi
I. Introduction . . . . .	1
Background . . . . .	1
Problem and Scope . . . . .	2
General Approach and Presentation . . . . .	3
II. Signal Processing-Two Dimensional Images . . . . .	5
III. Scattering from the Perfectly-Conducting Sphere in the Far Field . . . . .	13
Frequency Response of the Sphere . . . . .	13
Process to Obtain Backscattered Signal . . . . .	22
IV. Scattering from the Perfectly-Conducting Infinitely-Thin Circular Disk in the Far Field . . . . .	35
Sensitivity of Returns from the Circular Disk at Five Different Aspect Angles . . . . .	37
Backscattering from the Circular Disk at $1^{\circ}$ Off Normal . . . . .	38
Backscattering from the Circular Disk from $71^{\circ}$ , $72^{\circ}$ , $73^{\circ}$ , $74^{\circ}$ , and $75^{\circ}$ off Normal . . . . .	42
Backscattering from the Circular Disk at $89^{\circ}$ off Normal . . . . .	45
V. Conclusions, Findings, and Recommendations . . . . .	60
Conclusions . . . . .	60
Findings . . . . .	60
Recommendations . . . . .	61
Bibliography . . . . .	62
Appendix A: Explanation and Program Listing for the Frequency Response of the Sphere . . . . .	64

Appendix B: Explanation and Program Listings for the Frequency Response of the Far Field Scattering of the Perfectly-Conducting Infinitely-Thin Circular Disk . . . . .	71
Appendix C: Program Listing for the Fast Fourier Transform (FFT) . . . . .	103
Appendix D: Brief Review of the Selection of the FFT Parameters . . . . .	106
Vita . . . . .	110



### List of Figures

Figure	Page
1 Two Discrete Point Scatterers in the Far Field . . . . .	6
2 Bank of Matched Filters at Receiver . . . . .	9
3 Plot of Magnitude of Equation (13) . . . . .	11
4 Identification Process . . . . .	14
5 Frequency Response of the Scattering from the Sphere as a Function of $ka$ . . . . .	21
6 Incident Pulse of R-F at 10GHz . . . . .	23
7 Expanded View of Incident R-F Pulse . . . . .	24
8 Spectrum of Incident R-F Pulse . . . . .	25
9 Frequency Response of the Scattering of the Sphere . . . .	26
10 Real Part of Inverse Fourier Transform of the Frequency Response of the Scattering from the Sphere . . . . .	28
11 Real Part of Reflected Signal with R-F Pulse Incident . .	29
12 Video Pulse . . . . .	30
13 Spectrum of Video Pulse . . . . .	31
14 Real Part of Reflected Signal with Video Pulse Incident . . . . .	32
15 Geometry of Disk . . . . .	35
16 Frequency Response of the Scattering from the Circular Disk with $\theta_0=1^\circ$ . . . . .	39
17 Phase of the Frequency Response of the Scattering from the Circular Disk with $\theta_0=1^\circ$ . . . . .	40

18	Reflected Signal from Circular Disk with Video Pulse Incident at $1^\circ$ . . . . .	41
19	Reflected Signal from Circular Disk with R-F Pulse Incident at $1^\circ$ . . . . .	43
20	Frequency Response of the Scattering from the Circular Disk with $\theta_0=71^\circ$ . . . . .	46
21	Frequency Response of the Scattering from the Circular Disk with $\theta_0=72^\circ$ . . . . .	47
22	Frequency Response of the Scattering from the Circular Disk with $\theta_0=73^\circ$ . . . . .	48
23	Frequency Response of the Scattering from the Circular Disk with $\theta_0=74^\circ$ . . . . .	49
24	Frequency Response of the Scattering from the Circular Disk with $\theta_0=75^\circ$ . . . . .	50
25	Reflected Signal from the Circular Disk with R-F Pulse Incident at $71^\circ$ . . . . .	51
26	Reflected Signal from the Circular Disk with R-F Pulse Incident at $72^\circ$ . . . . .	52
27	Reflected Signal from the Circular Disk with R-F Pulse Incident at $73^\circ$ . . . . .	53
28	Reflected Signal from the Circular Disk with R-F Pulse Incident at $74^\circ$ . . . . .	54
29	Reflected Signal from the Circular Disk with R-F Pulse Incident at $75^\circ$ . . . . .	55
30	Frequency Response of the Scattering from the Circular Disk with $\theta_0=89^\circ$ . . . . .	56
31	Reflected Signal from the Circular Disk with R-F Pulse Incident at $89^\circ$ . . . . .	57
32	Reflected Signal from the Circular Disk with Video Pulse Incident at $89^\circ$ . . . . .	58

Abstract

The problem of identifying the perfectly conducting sphere and perfectly-conducting, infinitely-thin circular disk via the backscattered far field from the object with a known incident signal is examined. A method utilizing Synthetic Aperture radar principles to obtain slant range versus cross range image plots of the discrete point scatterers on an object is discussed. A computer program to calculate the far field backscattering from the sphere is developed. Another computer program, found in the literature, which calculates the far field backscattering from the infinitely-thin, circular disk, is extended for  $ka > 15$ . In the above programs, the backscattering is obtained as a function of frequency and treated as the frequency response of a linear system. The input to this system is five cycles of 10GHz R-F. The return signal is the output of the system and is obtained by using conventional linear system theory. Since a sphere is symmetric, the same return is obtained at all aspect angles. This return from the sphere is examined. However, returns from the circular disk are different at different aspect angles. These returns from the disk are examined.

PREDICTED MICROWAVE ELECTRO-MAGNETIC BACKSCATTERING  
RETURNS FOR SIMPLE REFLECTIVE TARGETS

I. Introduction

Background

With the use of Synthetic Aperture Radar processing, it is theoretically possible to obtain two-dimensional slant range versus cross range image plots of the locations of the discrete point scatterers on simple geometrical shapes. These plots, it is hoped, will show a rough outline of the object and will aid in its identification. As was stated in the Preface, an aircraft will be thought of as a collection of these simple geometrical objects whose image plots are known at different aspect angles.

The two simple geometrical shapes considered here are the sphere and the circular disk. The sphere is unique in the fact that at any aspect angle the return signal will always be the same. This is due to spherical symmetry. Thus, at any specific aspect angle, the image plot obtained from the sphere will be the same as the image plot obtained at any other aspect angle. As will be shown in Chapter II, to obtain cross range information, the return signal must change with changing aspect angle. Thus, for the sphere, only slant range information can be obtained. The circular disk, however, has the advantage of returning an aspect-dependent signal. Therefore, cross range information can be obtained for the disk. The circular disk also

possesses a known backscattered frequency response which is treated as the frequency response of a linear system. The backscattered signal is the output of the system and the incident signal is the input to the system. Although the backscattered signal from the sphere provides no cross range information, its backscattered signal can be obtained in the same way as the backscattered signal from the disk.

The method used for identification of the unknown shape is to compare the image obtained from the unknown shape to previously recorded images of different known shapes at known aspect angles. Hopefully, one of the known image plots will match up with the unknown plot. Taken that this is the case, the unknown object will be identified as the object which corresponds to the known matching image plot.

#### Problem and Scope

The specific problem studied in this thesis is how many distinct backscatter signature returns are obtained over a specified angle variation around the circular disk. The sphere is spherically symmetric and thus will backscatter the same return signal for all incident aspect angles. For this reason, the section on the sphere was used to show how the return signal was obtained by knowledge of the frequency response of the scattering. The return signal from the disk is obtained in the same manner. In this case, however, the frequency response of the scattering is aspect dependent.

Only the sphere and the circular disk are considered in depth since they are the only simple geometrical shapes for which the

frequency response of the far field scattering can be numerically calculated.

#### General Approach and Presentation

The general approach used in the research was to firstly understand how two-dimensional images of the discrete point scatterers on simple geometrical objects can be obtained. Synthetic Aperture Radar processing was seen to aid in the development of a slant range versus cross range image of the discrete point scatterers of the object.

Secondly, there had to be discovered if there were any geometrical shapes for which the backscattered field as a function of frequency was numerically calculable. Once the sphere and the circular disk were found to possess this quality, they were the only shapes concentrated on. As was alluded to in the background section, this backscattered field, as a function of frequency, was treated as the frequency response of a linear system. The incident signal is the input to the system and the backscattered signal is the output. For the sphere, only the backscattered signal at one aspect angle is examined since the sphere is symmetric. For the disk, however, the returns from different aspect angles were examined.

The point of obtaining the backscattered signals from the sphere and the disk is that, theoretically, they can now become the inputs to the processor to produce the two-dimensional plots. Unfortunately, because of time constraints, this could not be attempted.

The sequence of presentation of the main chapters in the-  
sis is:

- I. Introduction
- II. Signal Processing-Two Dimensional Images
- III. Scattering from the Perfectly Conducting  
Sphere in the Far Field
- IV. Scattering from the Perfectly Conducting,  
Infinitely-Thin Circular Disk in the Far Field
- V. Conclusions, Findings and Recommendations.

## II. Signal Processing Two-Dimensional Images

The processing used to obtain two-dimensional images of the scattering centers of objects in the far field is essentially Synthetic Aperture Radar processing (Ref 3, Ref 8). The following is an explanation of the processing used to obtain a two-dimensional image of two discrete point scatterers in the far field (Ref 7).

Figure 1 is representative of the situation of two discrete point scatterers in the far field. Scatterer 1 is at point one and scatterer 2 is at point two. Imagine, however, that instead of the scatterers moving around the radar, that the radar moves around the scatterers. The radar is at point three. With no loss in generality, let scatterer 1 be located at zero cross range and let scatterer 2 be located at cross range  $R_{c2}$ . Also let the distance from the radar to scatterer 1 be constant and be denoted  $R_0$ . Let the distance from the radar to scatterer 2 be varying and be denoted  $R$ . Also let  $\Delta\theta/2$  and  $\theta$  denote the angles shown in Figure 1. The angle  $\Delta\theta/2 \approx 0^\circ$  and  $\theta \approx 90^\circ$ . Therefore, the cross range axis is perpendicular to the line from point one to point three.

The two dimensional image that will be obtained will be a plot of slant range versus cross range. Slant range can be obtained easily by noting the time delay of the signal from the radar to each point scatterer and back to the radar. Therefore, if the time delay to the  $i^{th}$  scatterer and back is denoted  $\tau_i$ , the slant range  $R_{si}$  is



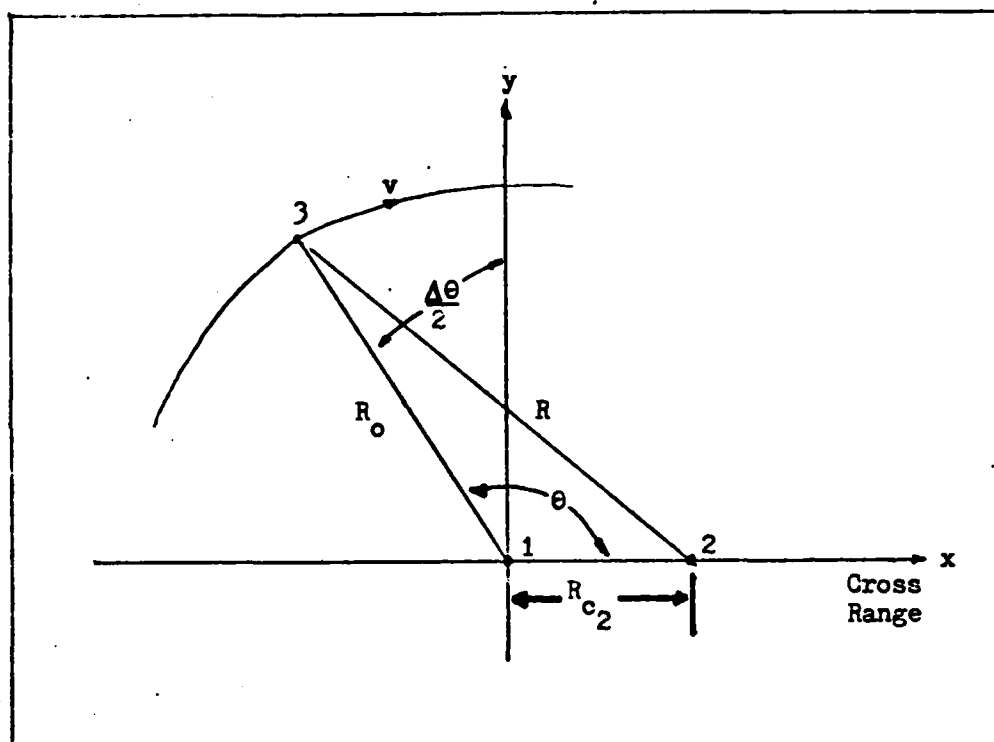


Figure 1. Two Discrete Point Scatterers in the Far Field

$$R_{s1} = \frac{c\tau_i}{2} \quad (1)$$

where

$c$  = speed of light in free space

As can be seen, the processing required to obtain slant range is quite simple in principle. On the other hand, the processing required to obtain cross range is more complicated. The cross range of scatterer 2 will be examined since the processing involved to obtain its cross range is representative of the processing involved to obtain the cross range of any discrete point scatterer on the target.

Assuming the transmitted signal from the radar is

$$s_T(t) = e^{j\omega_c t} \quad (2)$$

the signal received back from scatterer 2 will be

$$s_{R2}(t) = e^{j(\omega_c t - 2kR_{s2})} \quad (3)$$

where

$$k = \frac{2\pi}{\lambda} \quad (4)$$

Also, assuming that cross range values will be much smaller than slant range values,  $R_{s2}$  can be approximated by

$$R_{s2} \approx R_0 - R_{c2} \cos \theta \quad (5)$$

therefore,

$$s_{R2}(t) = e^{j\omega_c t} \cdot e^{-j2kR_0} \cdot e^{j2R_{c2} \cos \theta} \quad (6)$$

note that

$$\theta = \theta(t) = -\frac{v}{R_0}t + \frac{\pi}{2} + \frac{\Delta\theta}{2} \quad (7)$$

therefore

$$\begin{aligned} \cos \theta &= \cos \left( \frac{\pi}{2} + \frac{\Delta\theta}{2} - \frac{v}{R_0}t \right) \\ &= -\sin \left( \frac{\Delta\theta}{2} - \frac{v}{R_0}t \right) \end{aligned} \quad (8)$$

However, if  $\left( \frac{\Delta\theta}{2} - \frac{v}{R_0}t \right)$  is small, then

$$\cos \theta \approx \frac{v}{R_0}t - \frac{\Delta\theta}{2} \quad (9)$$

Therefore,

$$s_{Rs}(t) = e^{j\omega_c t} \cdot e^{-j2kR_0} \cdot e^{-jk\Delta\theta R_{c2}} \cdot e^{j\frac{2kR_{c2}v}{R_0}t} \quad (10)$$

At the receiver, there will be a bank of matched filters. Each of them will be matched to a specific cross range on either side of zero cross range. There will also be a filter matched to zero cross range.

The bank of matched filters is represented in Figure 2 where  $\Delta x$  is the cross range resolution and  $T$  is the dwell time. If there is a discrete scatterer at a cross range which is an integer multiple of  $\Delta x$ , the outputs of the filters matched to those

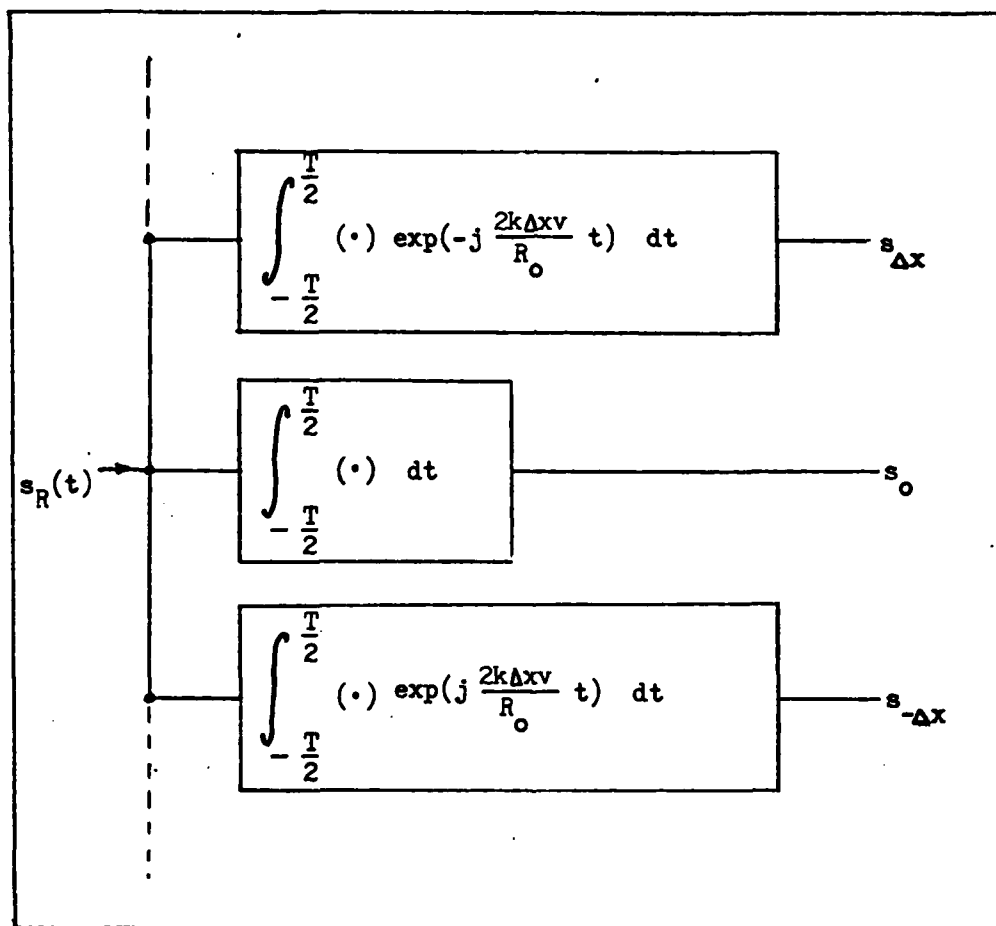


Figure 2. Bank of Matched Filters at Receiver

integer multiples will roll off with a  $\sin x/x$  distribution. For example, suppose there is a scatterer at cross range  $R_c$ . Therefore

$$s_{R2}(t) = e^{j\omega_c t} \cdot e^{-j2kR_0} \cdot e^{-jk\Delta\theta R_c} \cdot e^{j\frac{2kR_c v}{R_0} t} \quad (11)$$

The  $e^{j\omega_c t}$  term can be suppressed without loss in generality. Therefore,

$$s_{R2}(t) = e^{-j2kR_0} \cdot e^{-jk\Delta\theta R_c} \cdot e^{j\frac{2kR_c v}{R_0} t} \quad (12)$$

The output of the filter matched to  $R_{c2}$  would be

$$\begin{aligned} & \int_{-T/2}^{T/2} e^{-jk(2R_0 + \Delta\theta R_c)} \cdot e^{j\frac{2kR_c v}{R_0} t} \cdot e^{-j\frac{2kR_{c2} v}{R_0} t} dt \\ &= e^{-jk(2R_0 + \Delta\theta R_c)} \int_{-T/2}^{T/2} e^{j\frac{2kv}{R_0}(R_c - R_{c2})t} dt \\ &= e^{-jk(2R_0 + \Delta\theta R_c)} \frac{T \sin\left[\frac{2\pi v}{\lambda R_0}(R_c - R_{c2})T\right]}{\left[\frac{2\pi v}{\lambda R_0}(R_c - R_{c2})T\right]} \end{aligned} \quad (13)$$

A plot of the magnitude of the above equation as a function of  $R_c$  is shown in Figure 3. If  $R_c = R_{c2}$ , a maximum of  $T$  will be the output of the filter matched to  $R_{c2}$ . If  $R_c \neq R_{c2}$ , the output of that same filter will fall off as a  $\text{sinc}(x)$  distribution as a function of  $R_c$ .

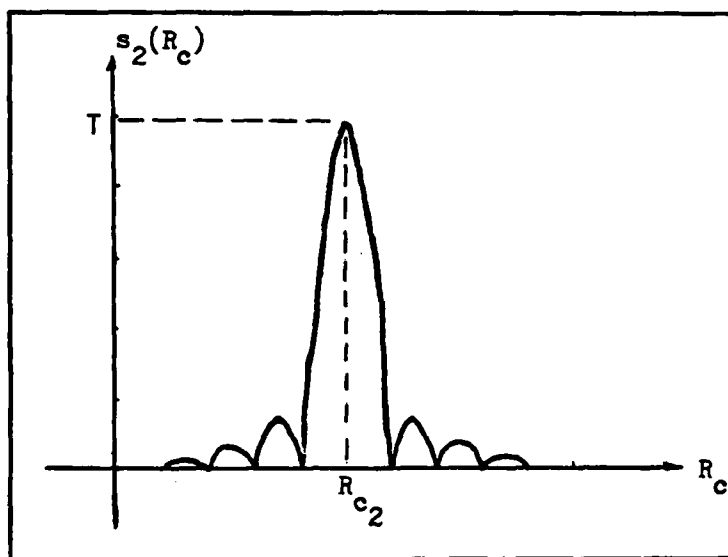


Figure 3. Plot of Magnitude of Equation (13)

To determine cross range resolution the criteria that will be used is that if the output of the filter matched to  $R_{c2}$  is above  $0.707T$  in magnitude, then there is a scatterer at cross range  $R_{c2}$ :  $\text{sinc}(x)=0.707$  at approximately 1.4 Rad. Therefore,  $\Delta R_c = R_c - R_{c2}$  must be found in Eq. (14) and multiplied by two to obtain  $\Delta x$ .

$$\frac{\sin\left[\frac{2\pi v}{\lambda R_0} \Delta R_c T\right]}{\left[\frac{2\pi v}{\lambda R_0} \Delta R_c T\right]} = 0.707 \quad (14)$$

Therefore

$$\frac{2\pi v}{\lambda R_0} \Delta R_c T \approx 1.4 \quad (15)$$

$$\Delta R_c = \frac{1.4 \lambda R_0}{2\pi v T} = \frac{0.22 \lambda R_0}{v T} \quad (16)$$

$$\Delta x = 2\Delta R_c = \frac{0.44 \lambda R_0}{v T} \quad (17)$$

After slant range and cross range of each scatterer are obtained, a two dimensional plot is made for a single target at a single aspect angle by locating these scatterers on a display.

### III. Scattering from the Perfectly Conducting Sphere in the Far Field

The electromagnetic backscattering from an object can be thought of as a linear system characterized by a frequency response which is the normalized backscattered E-field as a function of frequency (Ref 2:6). Therefore, in theory, the frequency spectrum of the backscattered signal can be obtained by multiplying the Fourier Transform of the incident signal by the frequency response of the system. The backscattered signal is the inverse Fourier Transform of its frequency spectrum. Thus, by conventional linear system theory, the scattered signal from an object can be obtained.

The reason for obtaining the backscattered signal from the object is that it can now be inputed to the processor described in Chapter II in order to obtain a two-dimensional, slant range vs. cross range image of the discrete point scatterers on the object. The process is represented in a block diagram in Figure 4.

#### Frequency Response of the Sphere.

The Mie series solution of the scattering from the sphere at a point  $p$  as a function of  $\omega$  is (Ref 14:142):

$$\bar{E}^S(p, \omega) = E_0 \sum_{n=1}^{\infty} (A_n \bar{M}_n + B_n \bar{N}_n) \quad (18)$$

where,



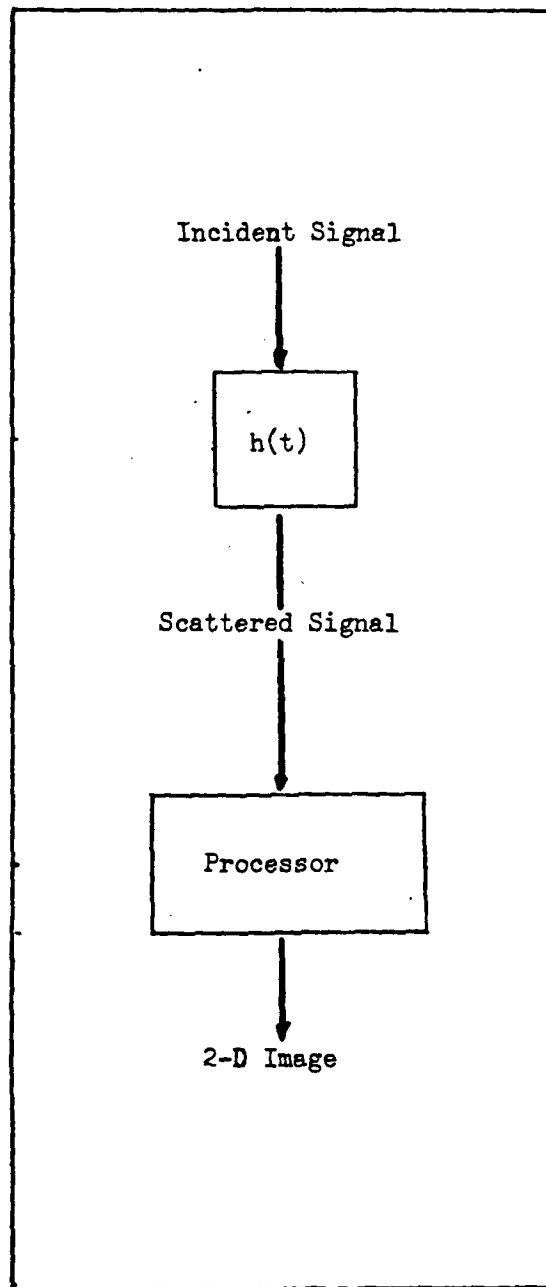


Figure 4. Identification Process

$$\begin{aligned} \text{Re } i_n &= \frac{1}{\sin \theta} h_n^1(kr) P_n^1(\cos \theta) (\cos \phi) \hat{\theta} \\ &\quad - h_n^1(kr) \frac{d}{d\theta} P_n^1(\cos \theta) (\sin \phi) \hat{\phi} \end{aligned} \quad (19)$$

$$\begin{aligned} \text{Im } i_n &= \frac{n(n+1)}{kr} h_n^1(kr) P_n^1(\cos \theta) (\cos \phi) \hat{r} \\ &\quad + \frac{1}{kr} \frac{d}{d(kr)} [kr h_n^1(kr)] \frac{d}{d\theta} P_n^1(\cos \theta) (\cos \phi) \hat{\theta} \\ &\quad - \frac{1}{kr \sin \theta} \frac{d}{d(kr)} [kr h_n^1(kr)] P_n^1(\cos \theta) (\cos \phi) \hat{\phi} \end{aligned} \quad (20)$$

$P_n^1(x) \triangleq$  Associated Legendre Polynomial of degree  $n$  and order 1.

$$= \frac{1}{2^n \cdot n!} \frac{d^n}{dx^n} (x^2 - 1)^n \quad (21)$$

$h_n^1(x) \triangleq$  Spherical Hankel Function of degree  $n$  and order 1.

$$= J_n(x) + jY_n(x) \quad (22)$$

$$Y_n(x) = \frac{J_n(x) \cos(n\pi) - J_{-n}(x)}{\sin(n\pi)} \quad (23)$$

n not an integer

$$A_n = -(-j)^n \frac{2n+1}{n(n+1)} \frac{j_n(ka)}{h_n^1(ka)} \quad (24)$$

$$B_n = (-j)^{n+1} \frac{2n+1}{n(n+1)} \frac{\frac{d}{d(ka)} [ka j_n(ka)]}{\frac{d}{d(ka)} [ka h_n^1(ka)]} \quad (25)$$

However, assuming  $r \gg a$ ,  $e^{-j\omega t}$  time dependence and using far field approximations, with "a" the radius of the sphere (Ref 14:143):

$$\vec{E}^S(p, \omega) = \frac{E_0 e^{jkr}}{kr} [\cos \phi S_1(\theta) \hat{\theta} - \sin \theta S_2(\theta) \hat{\phi}] \quad (26)$$

where

$$S_1(\theta) = \sum_{n=1}^{\infty} (-j)^{n+1} \left[ A_n \frac{P_n^1(\cos \theta)}{\sin \theta} + j B_n \frac{d}{d\theta} P_n^1(\cos \theta) \right] \quad (27)$$

$$S_2(\theta) = \sum_{n=1}^{\infty} (-j)^{n+1} \left[ A_n \frac{d}{d\theta} P_n^1(\cos \theta) + j B_n \frac{P_n^1(\cos \theta)}{\sin \theta} \right] \quad (28)$$

let,

$$F(\theta, \phi) \hat{\tau} = \cos \phi S_1(\theta) \hat{\theta} - \sin \phi S_2(\theta) \hat{\theta} \quad (29)$$

In the backscattering direction, approximations are made for  $F(0)$  for four regions: the low-frequency region, the lower resonance region, the upper resonance region and the high frequency region (Ref 14:146-150):

In the low frequency region ( $ka < 0.4$ ) the approximation for  $F(0)$  is (Ref 14:150):

$$F(0) = \frac{3}{2}(ka)^2 \quad (30)$$

therefore

$$rE^S(\omega) = \frac{E_0 e^{jkr}}{k} \cdot \frac{3}{2}(ka)^3 \quad (31)$$

In the lower resonance region ( $0.4 < ka < 0.8$ ) the approximation for  $F(0)$  is

$$\begin{aligned} F(0) = & \frac{3}{2}(ka)^3 \left[ 1 - \frac{5}{54}(ka)^2 + \frac{17}{900}(ka)^4 - \frac{6,651,923}{11,907,000}(ka)^6 \right] \\ & + j \frac{1}{2}(ka)^6 \left[ 1 + \frac{6}{5}(ka)^2 \right] \end{aligned} \quad (32)$$

$$= \sqrt{x^2 + y^2} e^{j \tan^{-1} \frac{y}{x}}$$

where

$$x = \frac{3}{2}(ka)^3 \left[ 1 - \frac{5}{54}(ka)^2 + \frac{17}{900}(ka)^4 - \frac{6,651,923}{11,907,000}(ka)^6 \right] \quad (33)$$

$$y = \frac{1}{2}(ka)^6 \left[ 1 + \frac{6}{5}(ka)^2 \right] \quad (34)$$

therefore

$$rE^s(\omega) = \frac{E_0 \sqrt{x^2 + y^2}}{k} \cdot e^{j(kr + \tan^{-1} \frac{y}{x})} \quad (35)$$

In the upper resonance region ( $0.8 < ka < 20$ ) the approximation for  $F(0)$  consists of a specular component,  $F^0(0)$ , added to a creeping wave term,  $F^C(0)$ . Therefore,

$$F(0) = F^0(0) + F^C(0) \quad (36)$$

The approximations of  $F^0(0)$  and  $F^C(0)$  are:

$$F^0(0) = \frac{ka}{2} e^{-j2ka} \left[ 1 - \frac{j}{2ka} \right] \quad (37)$$

$$F^C(0) = (ka)^{\frac{4}{3}} e^{j\pi(ka - \frac{7}{6})} \left[ 1.357588 + \right. \\ \left. + (0.741196 + j1.283788)(ka)^{-\frac{2}{3}} \right].$$

$$\begin{aligned}
& \cdot \exp [-(ka)^{\frac{1}{3}}(2.200002 - j1.270172) \\
& + (ka)^{-\frac{1}{3}}(0.445396 + j0.257150)] \\
& + [0.695864 + (0.964654 + j1.670829)(ka)^{-\frac{2}{3}}] \cdot \\
& \cdot \exp [-(ka)^{\frac{1}{3}}(7.014224 - j4.049663) \\
& - (ka)^{-\frac{1}{3}}(0.444477 + j0.256619)] \\
& - [0.807104 + (0.798821 + j1.383598)(ka)^{-\frac{2}{3}}] \cdot \\
& \cdot \exp [-(ka)^{\frac{1}{3}}(5.048956 - j2.915016) \\
& - (ka)^{-\frac{1}{3}}(0.312321 + j0.180319)] \tag{38}
\end{aligned}$$

let

$$F^0(0) + F^C(0) = \sqrt{x^2 + y^2} e^{\tan^{-1} \frac{y}{x}} \tag{39}$$

where

$$x = \operatorname{Re}\{F^0(0) + F^C(0)\} \tag{40}$$

$$y = \text{Im}\{F^0(0) + F^c(0)\} \quad (41)$$

therefore

$$rE^S(\omega) = E_0 \frac{\sqrt{x^2+y^2}}{k} e^{j(kr + \tan^{-1} \frac{y}{x})} \quad (42)$$

In the high frequency region ( $ka > 20$ ) the approximation for  $F(0)$  is

$$F(0) = -\frac{1}{2}ka e^{-j2ka} \quad (43)$$

therefore

$$\begin{aligned} rE^S(\omega) &= -\frac{E_0 ka}{2k} e^{-j[kr - 2ka]} \\ &= -\frac{E_0 a}{2} e^{j[kr - 2ka]} \end{aligned} \quad (44)$$

A computer program is listed in Appendix A which calculates the frequency response of the far field E-field backscattering from a perfectly conducting sphere. To give an idea of the shape of the backscattered E-field frequency response, a sample of the response is shown in Figure 5 as a function of  $ka$  up to  $ka=30$ . Since the scattering from the sphere is aspect independent, the response shown in Figure 5 is valid at any aspect angle.

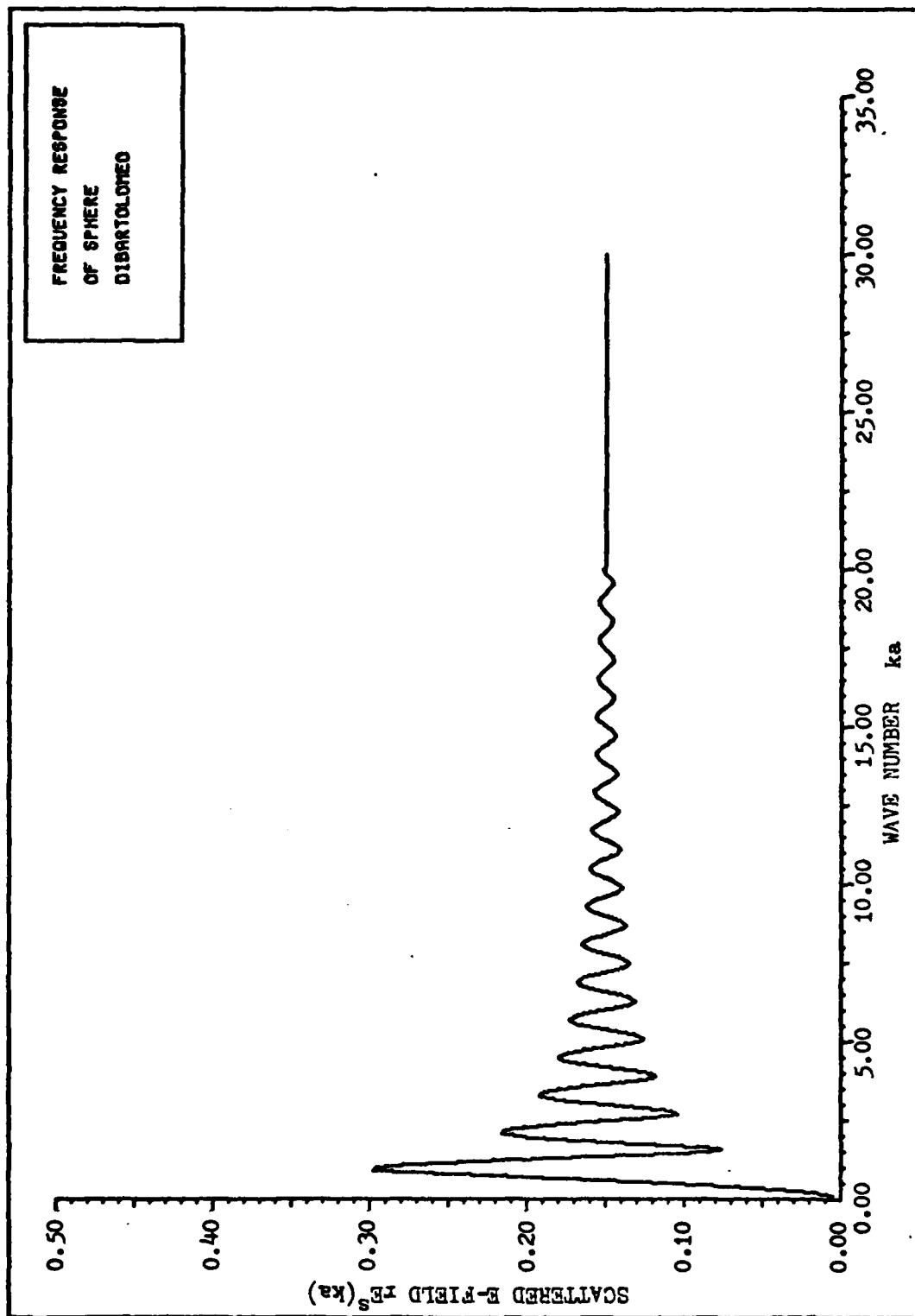


Figure 5. Frequency Response of the Scattering from the Sphere as a function of  $ka$



The facility for obtaining Fourier Transforms and Inverse Fourier Transforms in this thesis is the Fast Fourier Transform (FFT). An explanation as well as a listing of the computer program for the FFT is provided in Appendix C. A review of the proper selection of FFT parameters is provided in Appendix D.

#### Process to Obtain Backscattered Signal

The incident signal used is a pulse of five cycles of 10GHz R-F. The selection of the proper way to sample the incident pulse is explained in Appendix D. From Appendix D, the pulse is shown to be sampled ten times each cycle, or fifty times each pulse. The pulse is shown in Figure 6 centered at  $t=0$ . The pulse is shown split in two parts. One part is at  $t=0$  while the other part is at  $t=10.24$  nsec. This is done because the FFT cannot see negative time. The FFT assumes periodicity of the time signal and its frequency spectrum. Therefore, the pulse would repeat around  $t=10.24$  nsec (NOTE: 1024 samples, samples are 0.01 nsec apart). An expanded view of the left half of the pulse is shown in Figure 7. The spectrum of the pulse is shown in Figure 8. Note that the spectrum is simply a  $\text{sinc}(x)$  function shifted in frequency and centered on 10GHz. As outlined in Appendix D, the frequency response of any object must go out to the 250th sample, with  $\Delta f$  between samples being 97,656,000Hz, to fully enclose the appreciable frequency components of the incident 10GHz pulse. The frequency response for the sphere is shown in Figure 9, with the left half frequency reversed, conjugated and shifted up in frequency. The conjugation occurs, because when this frequency response is inverse transformed, the result should be a real impulse response.

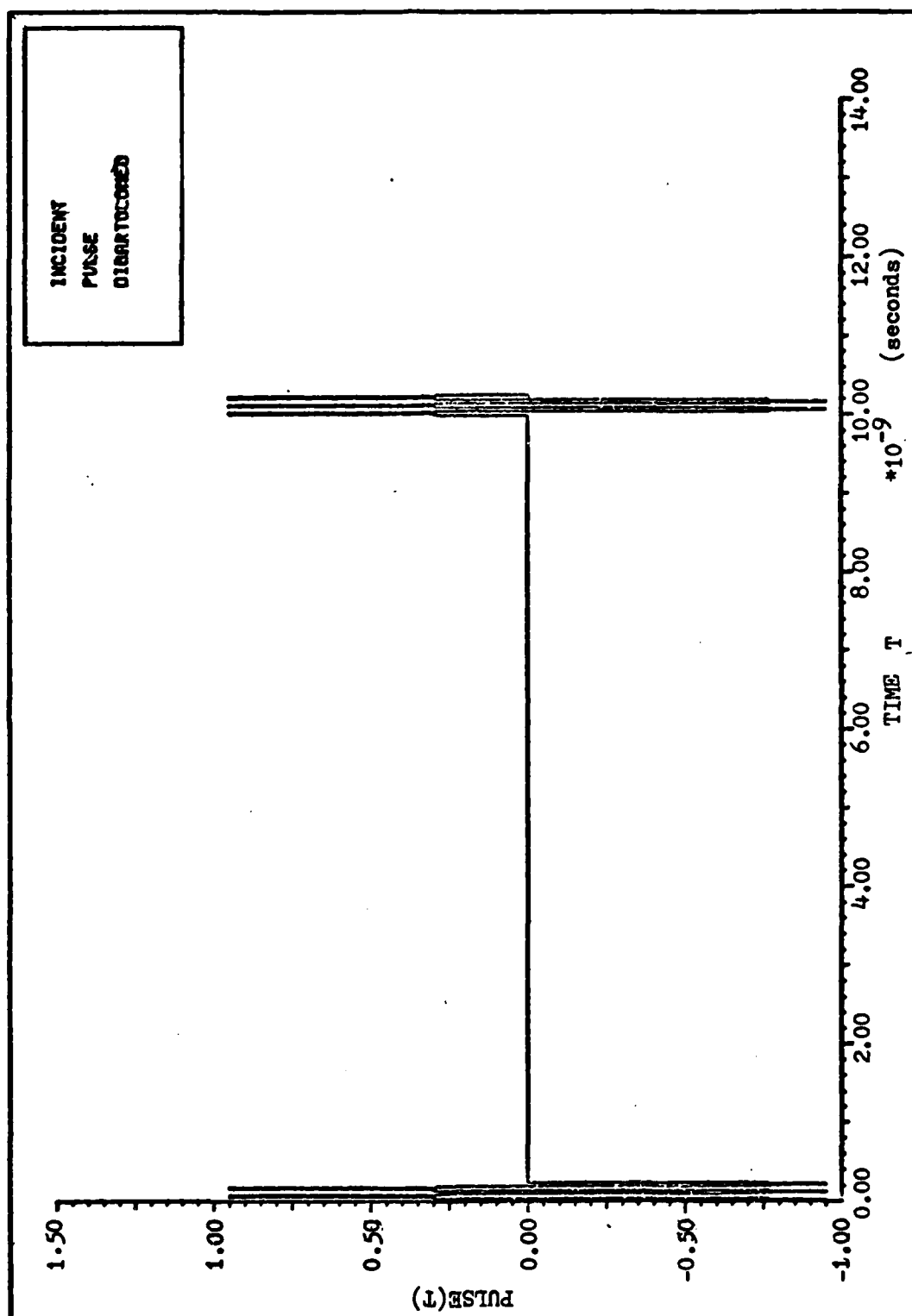


Figure 6. Incident Pulse of R-F at 10 GHz

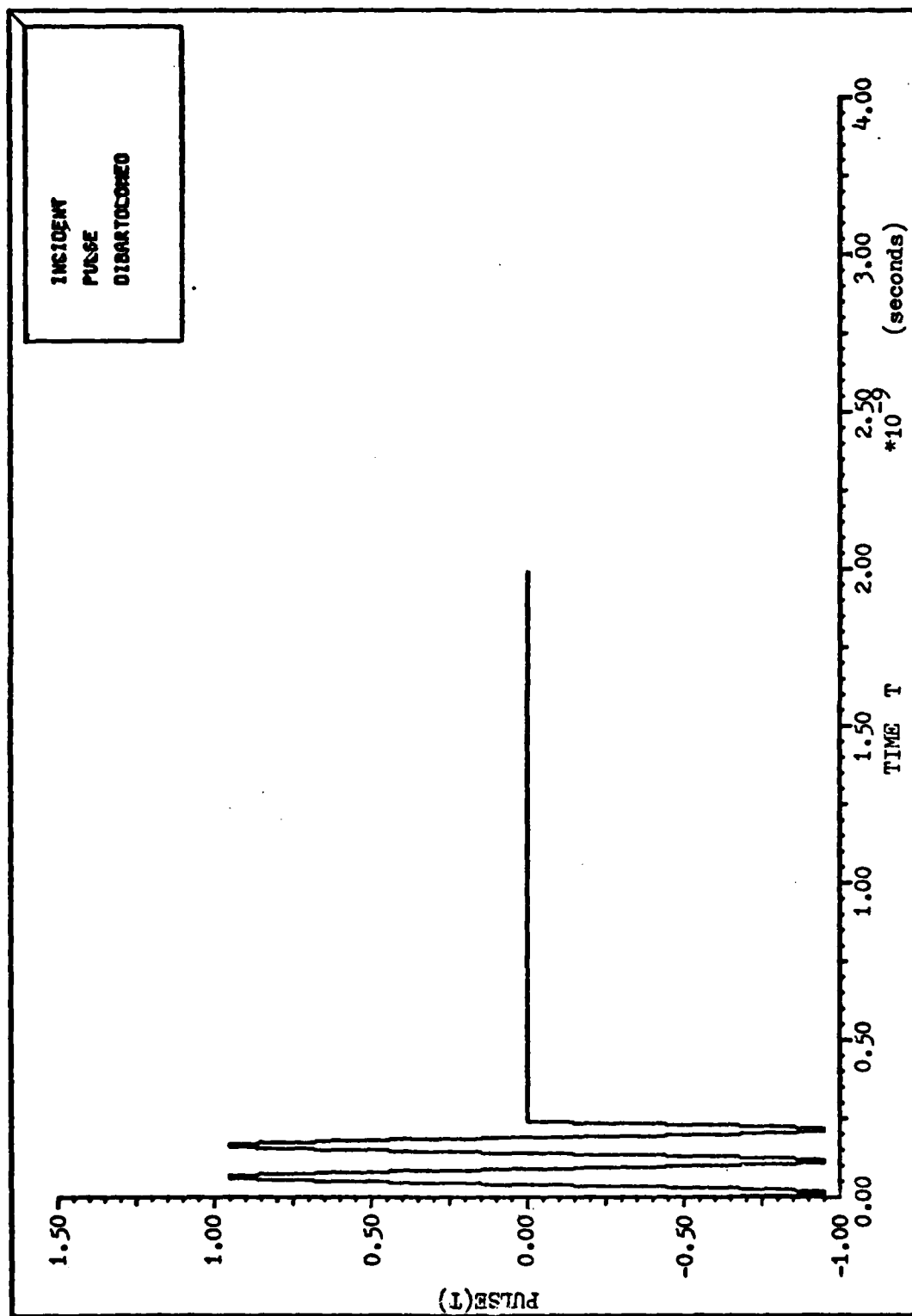


Figure 7. Expanded View of Incident R-F Pulse

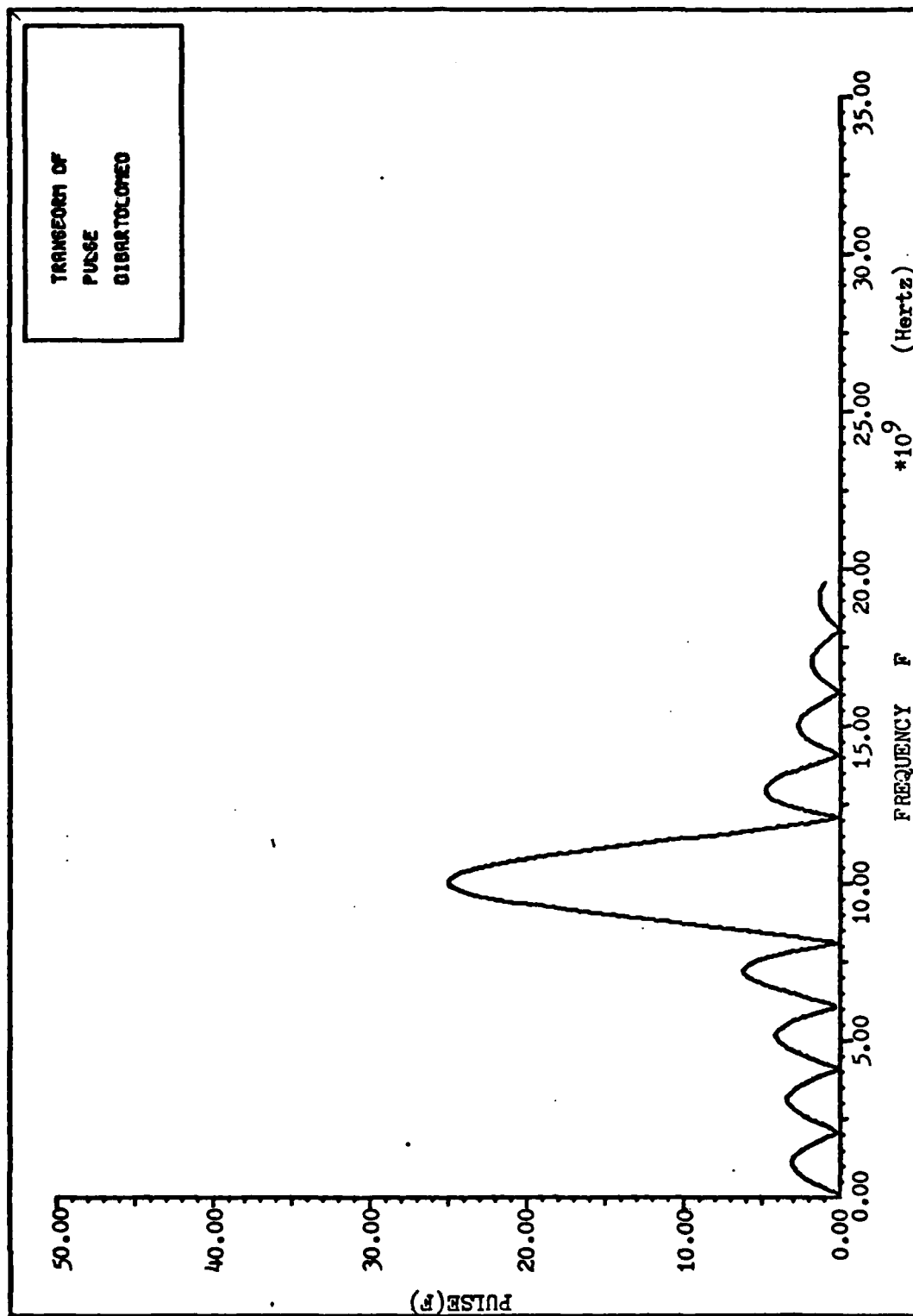


Figure 8. Spectrum of Incident R-F Pulse

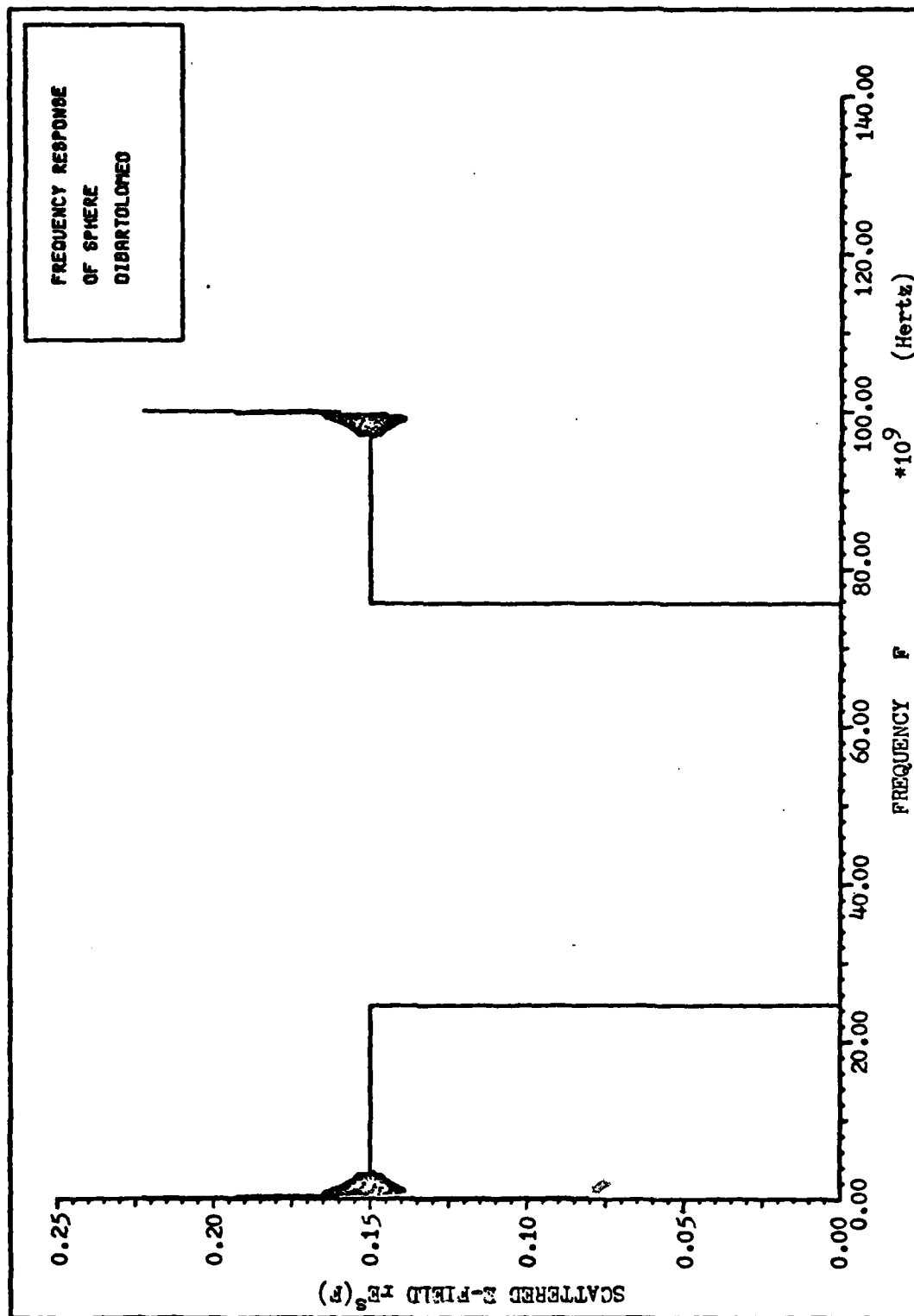


Figure 9. Frequency Response of the Scattering of the Sphere

It will be real if the frequency response has even magnitude and odd phase. The real part of the inverse Fourier Transform of the frequency response in Figure 9, is shown in Figure 10.

In Figure 11 is shown the real part of the result that is obtained when the frequency response of the sphere is multiplied by the Fourier Transform of the 10GHz incident pulse and inverse Fourier Transformed. Figures 12-14 show the same process done on a video pulse of the same width as the 10GHz pulse. Figure 12 is the video pulse with Figure 13 its spectrum, and Figure 14 the real part of the back-scattered signal. Figures 11 and 14 will be compared. Because the video pulse has frequency components down very low in frequency where the frequency response of the sphere is oscillatory, the response obtained by linearly convolving this pulse with the impulse response of the sphere is not exactly the video pulse returned as is shown in Figure 14. However, the appreciable frequency components of the frequency spectrum of the 10GHz pulse are out in frequency where the frequency response of the sphere is essentially flat. Therefore, as shown in Figure 11, the 10GHz pulse is returned almost intact with only a time delay. The slight negative response at the end of the graph in Figure 11 also holds much significance. This is the return from the incident signal creeping around the sphere. The significant point is that as the incident pulse gets shorter and shorter in duration, more and more physical features of the object that the pulse is illuminating will become noticeable. In the case of the sphere, there are only two returns; the specular return and the creeping wave return. An unfortunate result is

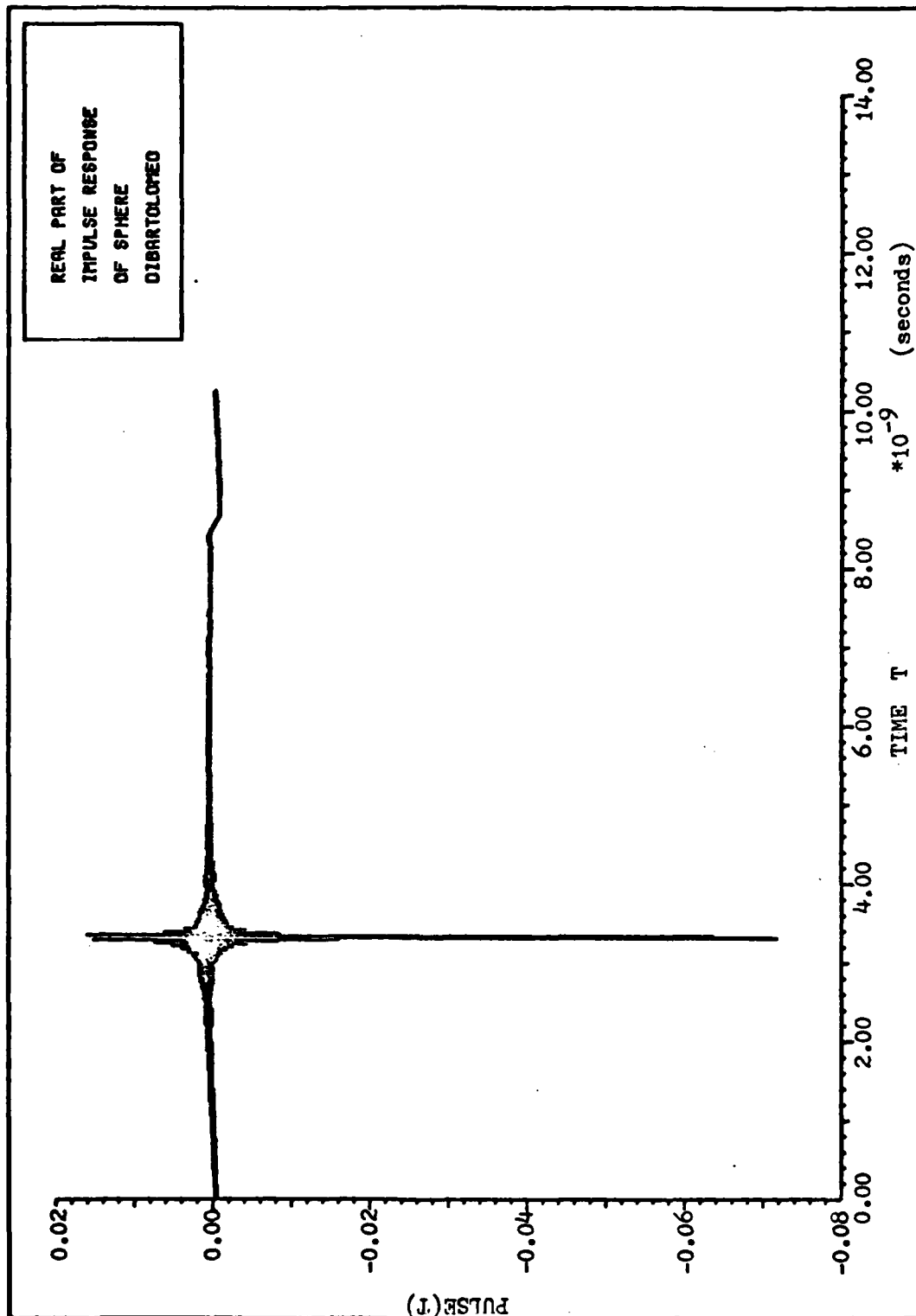


Figure 10. Real Part of Inverse Fourier Transform of the Frequency Response of the Scattering from the Sphere

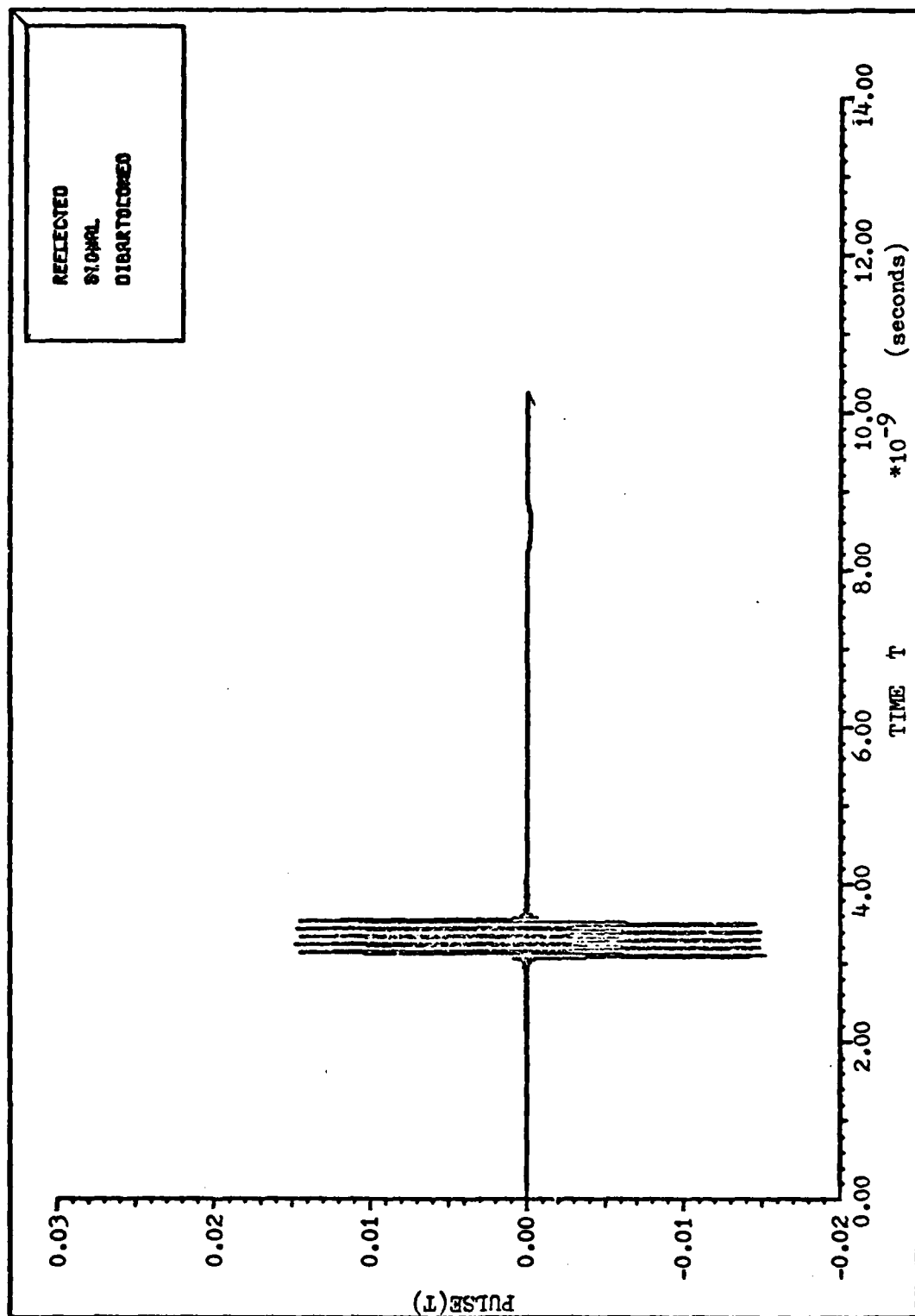


Figure 11. Real Part of Reflected Signal with R-F Pulse Incident



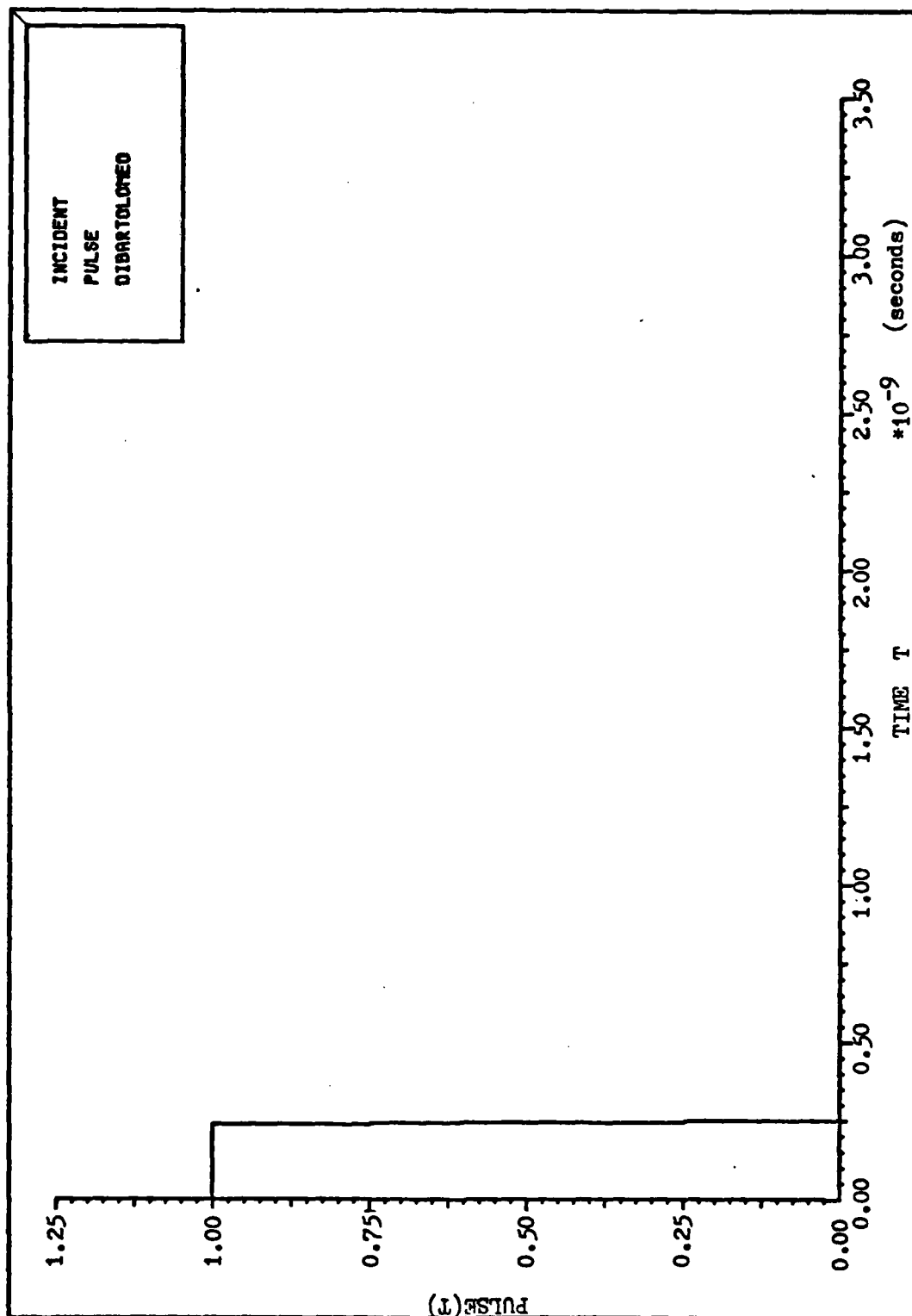


Figure 12. Video Pulse

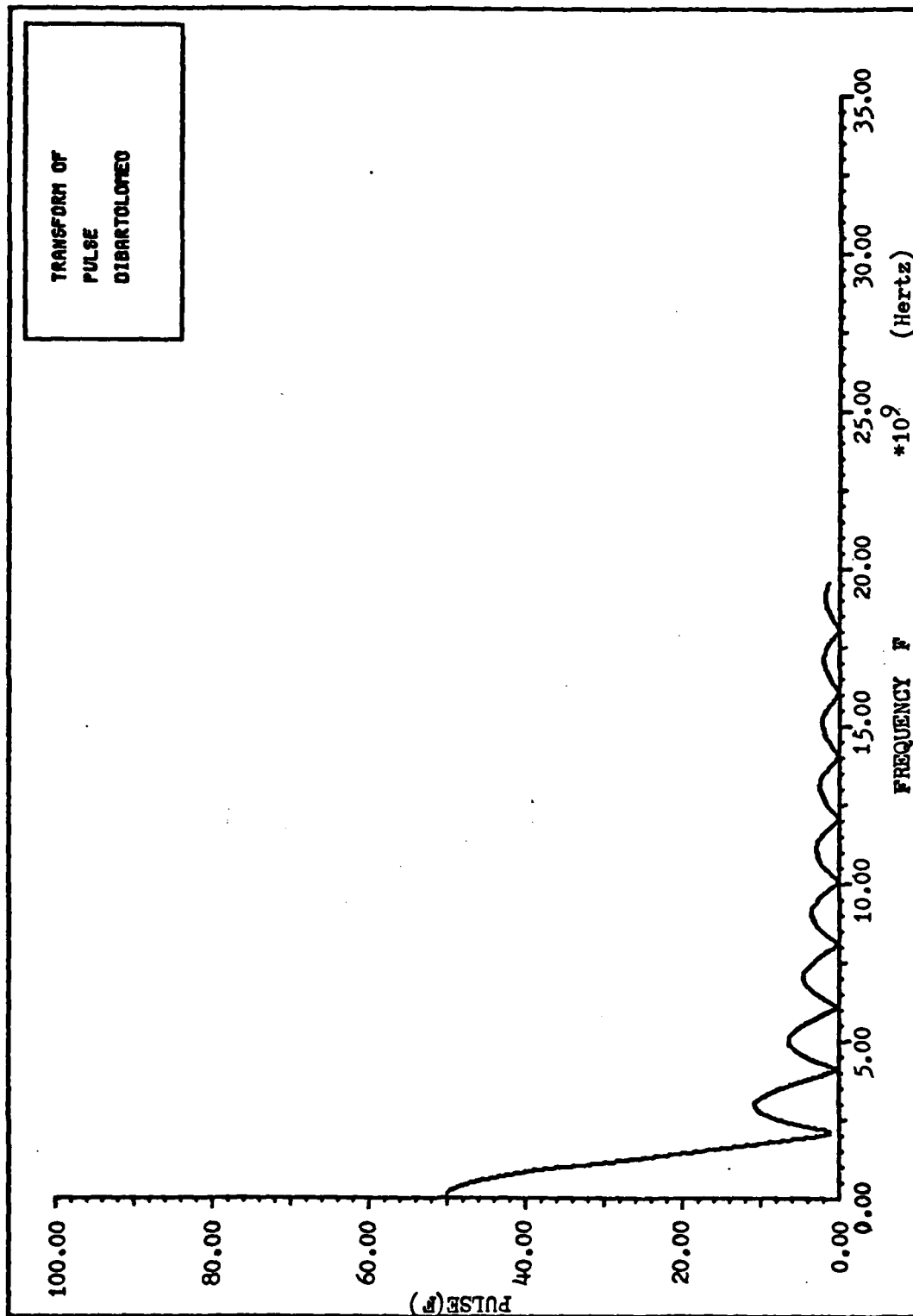


Figure 13. Spectrum of Video Pulse

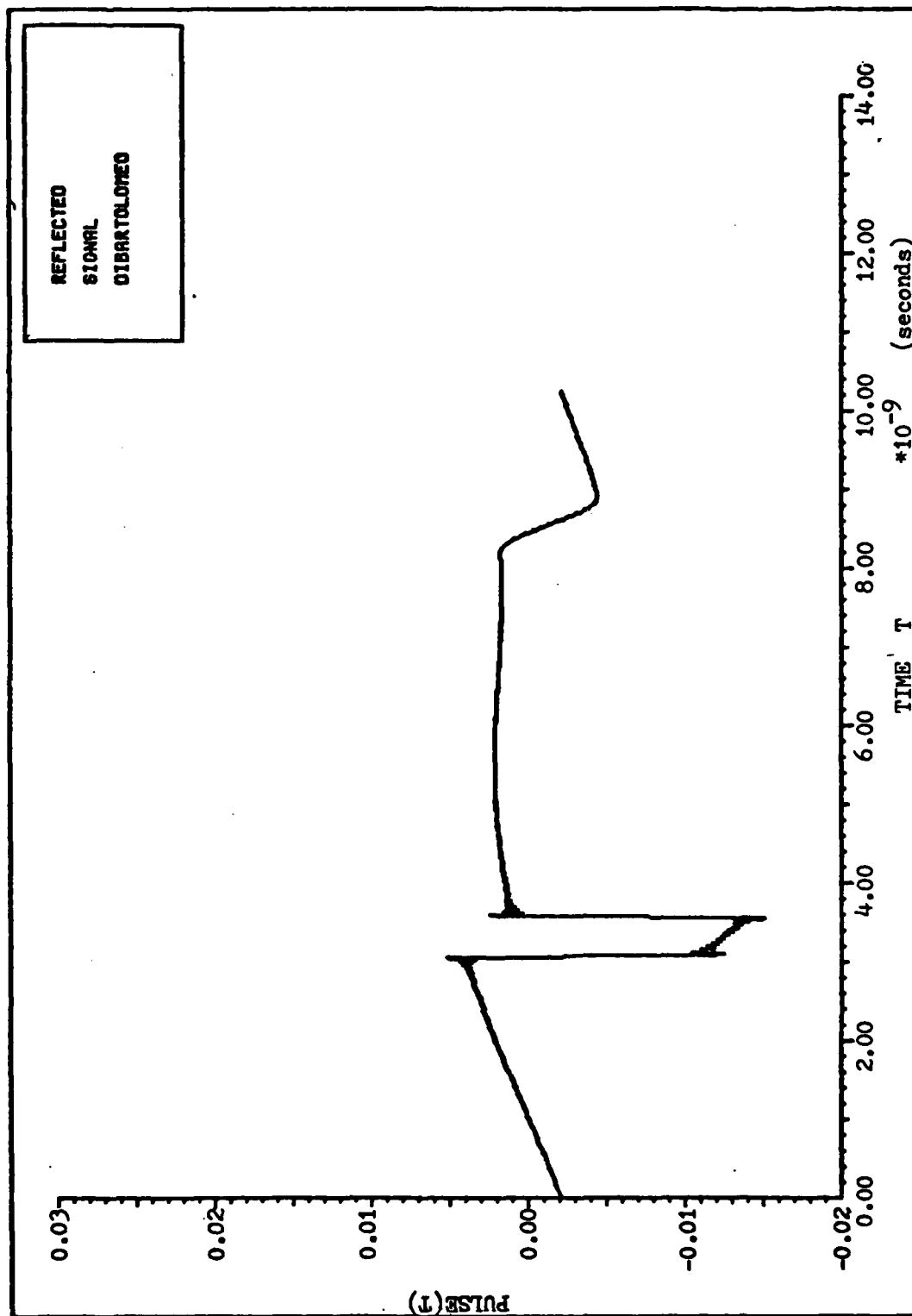


Figure 14. Real Part of Reflected Signal with Video Pulse Incident

that as the frequency of the pulse is increased, the creeping wave term will become less apparent. So only the specular return will be noticeable. At lower frequencies, the creeping wave return will become more apparent. Refer back to Eq. (17) in Chapter II, written here for convenience.

$$\Delta x = 2\Delta R_c = \frac{0.44\lambda R_0}{vT} \quad (17)$$

As the frequency is decreased in order to notice the creeping wave return, the wave length,  $\lambda$ , will be increased and thus increase  $\Delta x$  and thus decrease the cross range resolution. As the frequency is increased in order to decrease  $\lambda$  and  $\Delta x$  and therefore increase the cross range resolution of the discrete point scatterers, the creeping wave return will disappear. Thus, for all practical purposes, only the specular return will be detected.

#### IV. Scattering from the Perfectly Conducting, Infinitely-Thin Circular Disk in the Far Field

As was stated in the Introduction, the sphere, until recently was the only geometrical object for which numerical scattering results could be obtained. Although the rigorous eigenfunction solution of the scattering from the thin circular disk has been available in the literature for some time (Ref 1), only limited numerical results have been available due to the complexity of the numerical computations involved. However, in February 1979, a report out of the Electrosiences Laboratory at the Ohio State University was published for the Department of the Navy, Office of Naval Research, which made available the rigorous eigenfunction solution of the plane wave scattering problem from the infinitely-thin circular disk (Ref 9). The author, Dr. D. B. Hodge, develops a computer program to obtain the far field scattering from the thin metallic disk. The program allows an incident plane wave of arbitrary incidence angle and polarization and calculates the far field radar cross section as well as the scattered E-field. The geometry of the disk is shown in Figure 15. The direction of the incident wave is specified by  $k^i$  and  $\theta_0$ . The incident direction is restricted to the x-z plane. Because of circular symmetry, this is no restriction. The angle " $\alpha$ " is the polarization angle. When  $\alpha=0^\circ$ , the incident field is parallel to the x-z plane. When  $\alpha=90^\circ$ , the incident field is perpendicular to the x-z plane. The radius of the disk is "a." While the incident

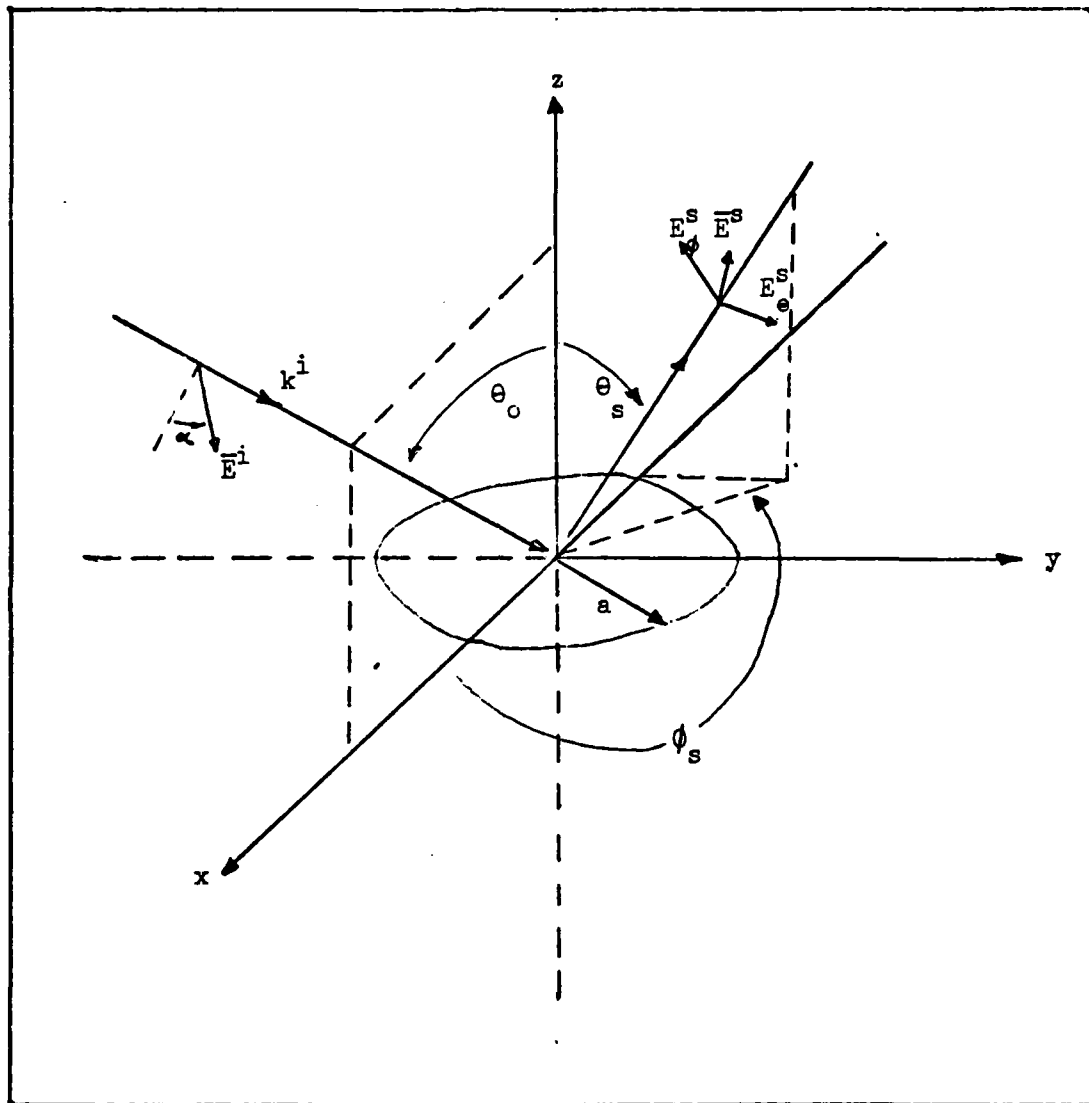


Figure 15. Geometry of Disk

field is restricted to the x-z plane, the scattered field can be in any direction specified by  $\theta_s$  and  $\phi_s$ . However, since backscattering is only considered,  $\theta_s = \theta_0$  and  $\phi_s = 0$ . A listing of Dr. Hodge's program is contained in Appendix B along with a sample I/O and an explanation of how to input values to the program.

The program is valid up to  $ka=15$ . A high-frequency approximation was used to extend the program past  $ka=15$ . This approximation is a modification of the Sommerfield-MacDonald technique by Ufimitsev (Refs 16 and 17). For  $0^\circ < \theta_0 < 90^\circ$  and  $\theta_s = \theta_0$ , the theta component of the E-field normalized by the radius of the disk and divided by two is (Ref14:514-515).

$$rE\theta_N^S \cong -j \left[ \left( A_1^2(\theta) \left( \frac{1-\sin \theta_0}{2 \sin \theta_0} \right) + A_2^2(\theta) \left( \frac{1+\sin \theta_0}{2 \sin \theta_0} \right) \right) \cdot \right. \\ \cdot J_1(2ka \sin \theta_0) - j \left[ A_1^2(\theta) \left( \frac{1-\sin \theta_0}{2 \sin \theta_0} \right) \right. \\ \left. \left. - A_2^2(\theta) \left( \frac{1+\sin \theta_0}{2 \sin \theta_0} \right) \right] J_2(2ka \sin \theta_0) \right] \quad (45)$$

where

$$A_{\frac{1}{2}}(\theta) = \sqrt{2} \exp \left[ -j\frac{\pi}{4} \right] \left[ C \left[ 2\sqrt{\frac{2ka}{\pi}} \left| \cos\left(\frac{\pi}{4} \mp \frac{\theta_0}{2}\right) \right| \right] \right. \\ \left. + jS \left[ 2\sqrt{\frac{2ka}{\pi}} \left| \cos\left(\frac{\pi}{4} \mp \frac{\theta_0}{4}\right) \right| \right] \right] \quad (46)$$

$$C(\eta) = \int_0^\eta \cos \frac{\pi}{2} t^2 dt \quad (47)$$

$$S(\eta) = \int_0^\eta \sin \frac{\pi}{2} t^2 dt \quad (48)$$

$$\sigma_{||} = 4\pi r^2 |E_\theta|^2 \quad (49)$$

To limit the extent of the programs listed in Appendix B,  $\alpha=0$ , in Figure 15. Since, such is the case, only  $E_\theta^S$  need be considered since  $E_\phi^S$  will be zero. The program used to implement the above equations for  $E_\theta^S$  and  $\sigma_{||}$  is in Appendix B with an output following the first program. The computer program that incorporates the extension to the first program is the third program listed in Appendix B.

#### Sensitivity of Returns from the Circular Disk at Five Different Aspect Angles

The following discussion is divided into three sections. The first section concerns backscattering from the circular disk at



1° off normal incidence. The second section is a sensitivity study of the backscattering from the disk at 71°, 72°, 73°, 74°, and 75° off normal incidence. The last section examines backscattering from the disk at 89° off normal incidence.

#### Backscattering from the Circular Disk at 1° Off Normal.

Figure 16 shows the frequency response of the backscattered E-field from the circular disk at 1° off normal. The large oscillatory portion of the response is the backscattering in accordance with the physical optics model. If the frequency of the incident E-field is constant and is directed at a specific aspect angle off the normal of the disk, a lobing pattern, quite similar to an antenna lobing pattern, will be created with the maximum of the major lobe pointed at the same angle off normal as the incident E-field and 180° around the disk from the incidence direction. This pattern is the magnitude of the scattered E-field as a function of aspect angle. As the frequency is increased and the incidence angle is kept constant, the beam widths of the lobes of the lobing pattern become smaller and consequently maxima and minima of the lobing pattern pass by the fixed incident angle resulting in the oscillatory nature of the frequency response of the backscattered E-field. The oscillatory response riding on the physical optics response is due to diffraction off the edges of the disk. If this response were to be plotted out by itself, it would be seen to be oscillatory in nature although of much smaller magnitude than the physical optics response. Figure 17 is a plot of the phase of the frequency response whereas Figure 16 is a plot of the magnitude of the frequency response. Figure 18

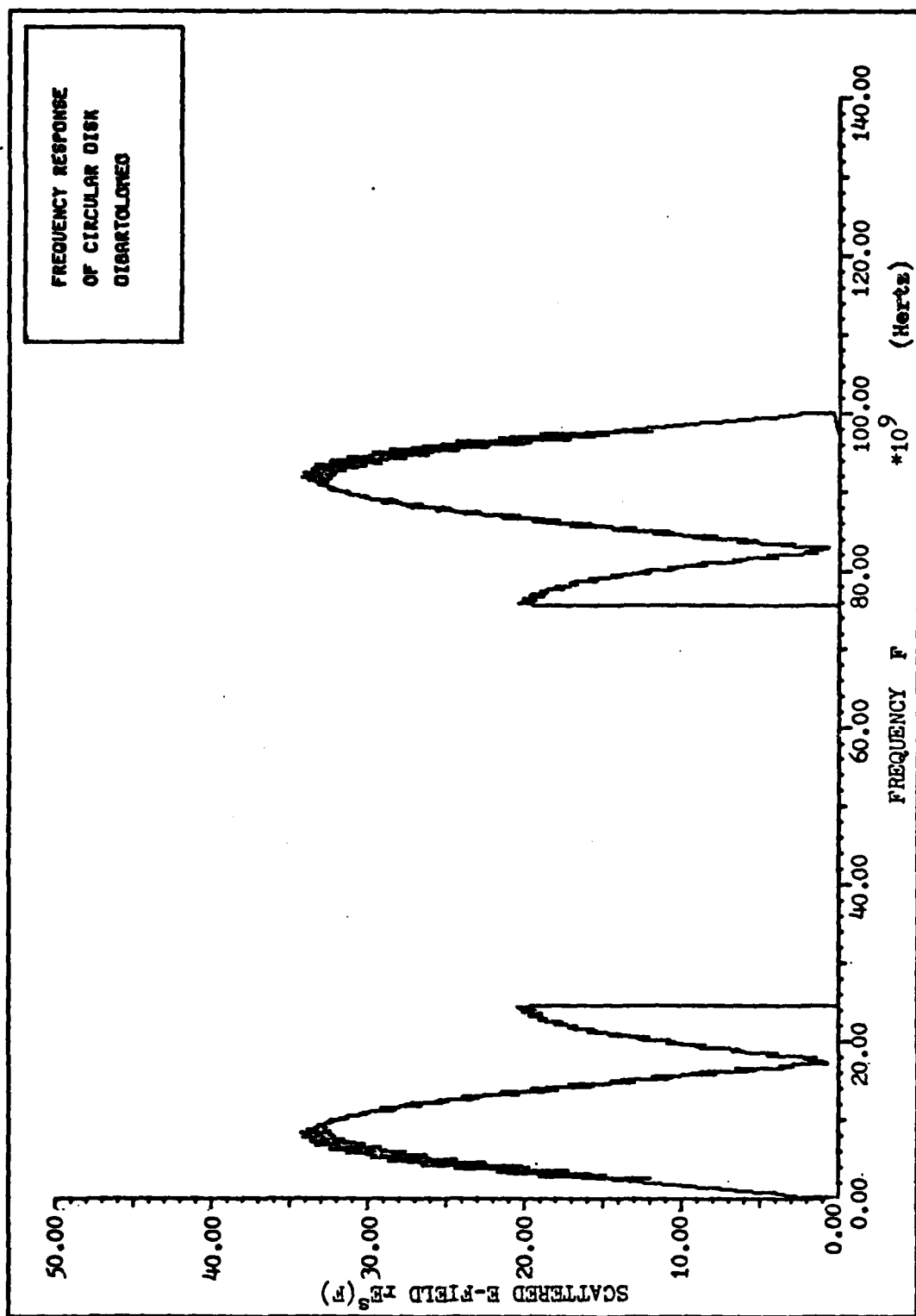


Figure 16. Frequency Response of the Scattering from the Circular Disk with  $\theta_0=1^\circ$

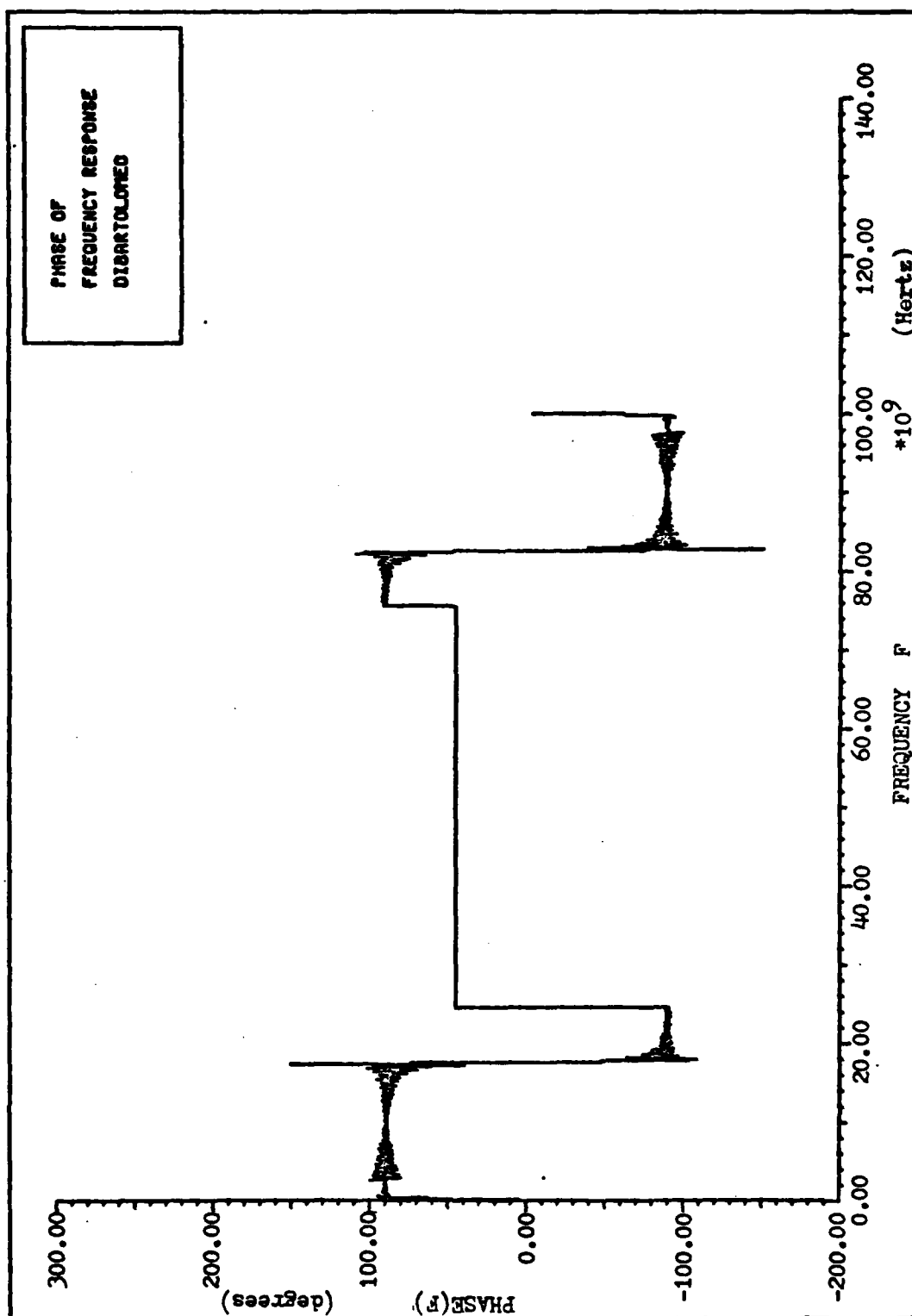


Figure 17. Phase of the Frequency Response of the Scattering from the Circular Disk with  $\theta_0=1^\circ$

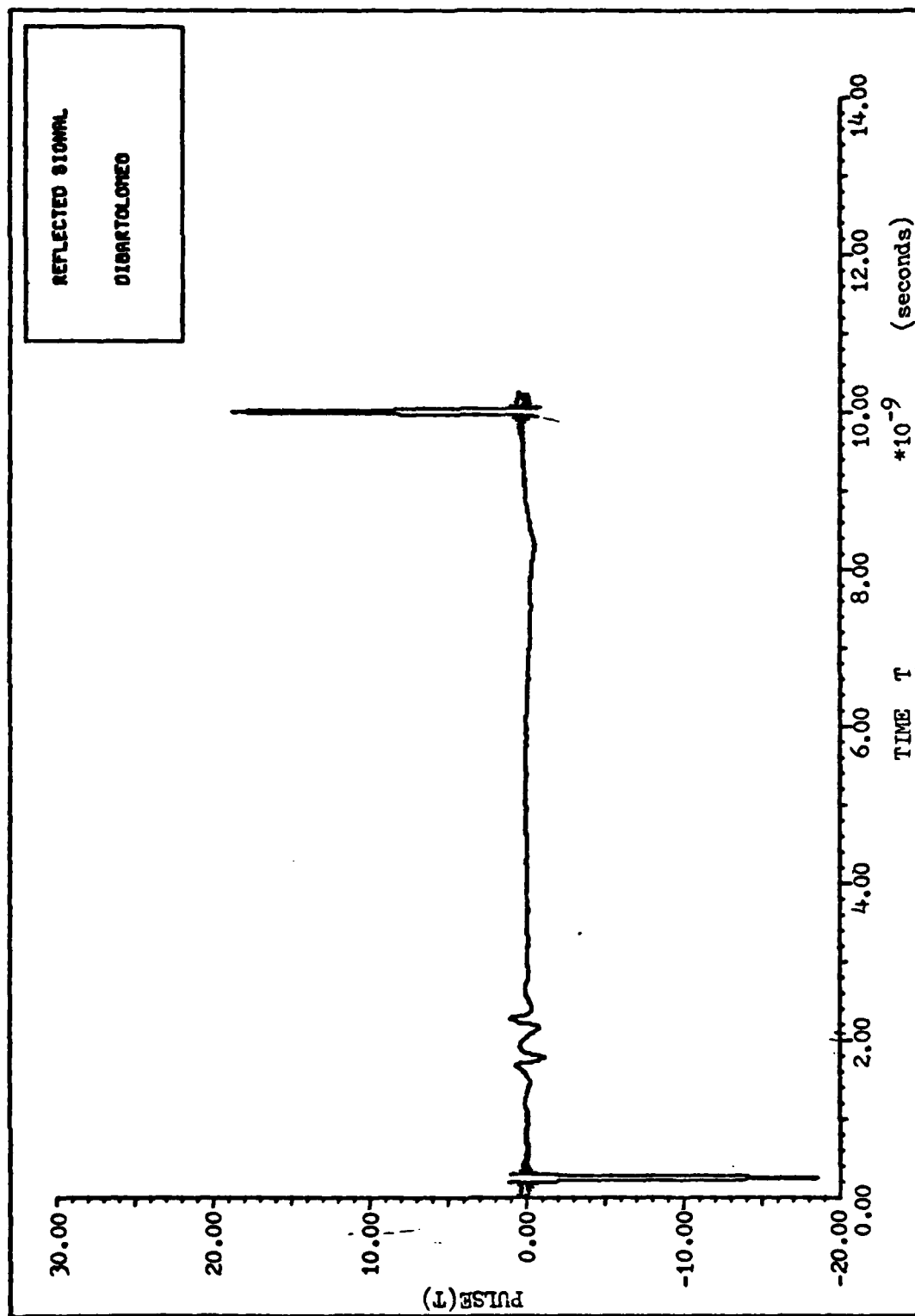


Figure 18. Reflected Signal from Circular Disk with Video Pulse Incident at  $1^\circ$

is a plot of the backscattered signal at  $1^\circ$  off normal incidence when a video pulse 0.5 nsec wide with a pulse repetition frequency of 100MHz is incident. This "impulse response" is seen to have a doublet at the leading edge of the response. This contribution is due to a term proportional to  $ka$  in the scattered electric field and is predicted by both physical optics and the geometrical theory of diffraction (Ref 10:559). Although the response in Figure 18 is not truly the impulse response since the incident pulse is not an impulse but a video pulse of width 0.5 nsec, the response obtained closely resembles results obtained in the literature (Ref 10:p.559). The backscattered signals obtained when a pulse of five cycles of 10GHz R-F is incident at  $1^\circ$  is shown in Figure 19. The effect of the doublet is still seen as the phase of the signal immediately to the right of  $t=0$  has a phase of  $180^\circ$  relative to the incident signal shown in Figure 6.

Backscattering from the Circular Disk from  $71^\circ$ ,  $72^\circ$ ,  $73^\circ$ ,  $74^\circ$ , and  $75^\circ$  off Normal. As was stated in the Introduction, the specific problem studied in this thesis is how many distinct backscatter signature returns are obtained over a specified angle variation around the circular disk. The vehicle for accomplishing this was to find the angle for which the backscatter frequency response had the lowest minimum at 10GHz (remember the incident pulse is five cycles of 10GHz R-F). The angle is  $73^\circ$  and was obtained with the use of the combination program or 3rd program in Appendix B. The angle variation, mentioned above, was centered on  $73^\circ$ . This variation in angle was from  $71^\circ$  to  $75^\circ$  in one degree increments. The frequency responses

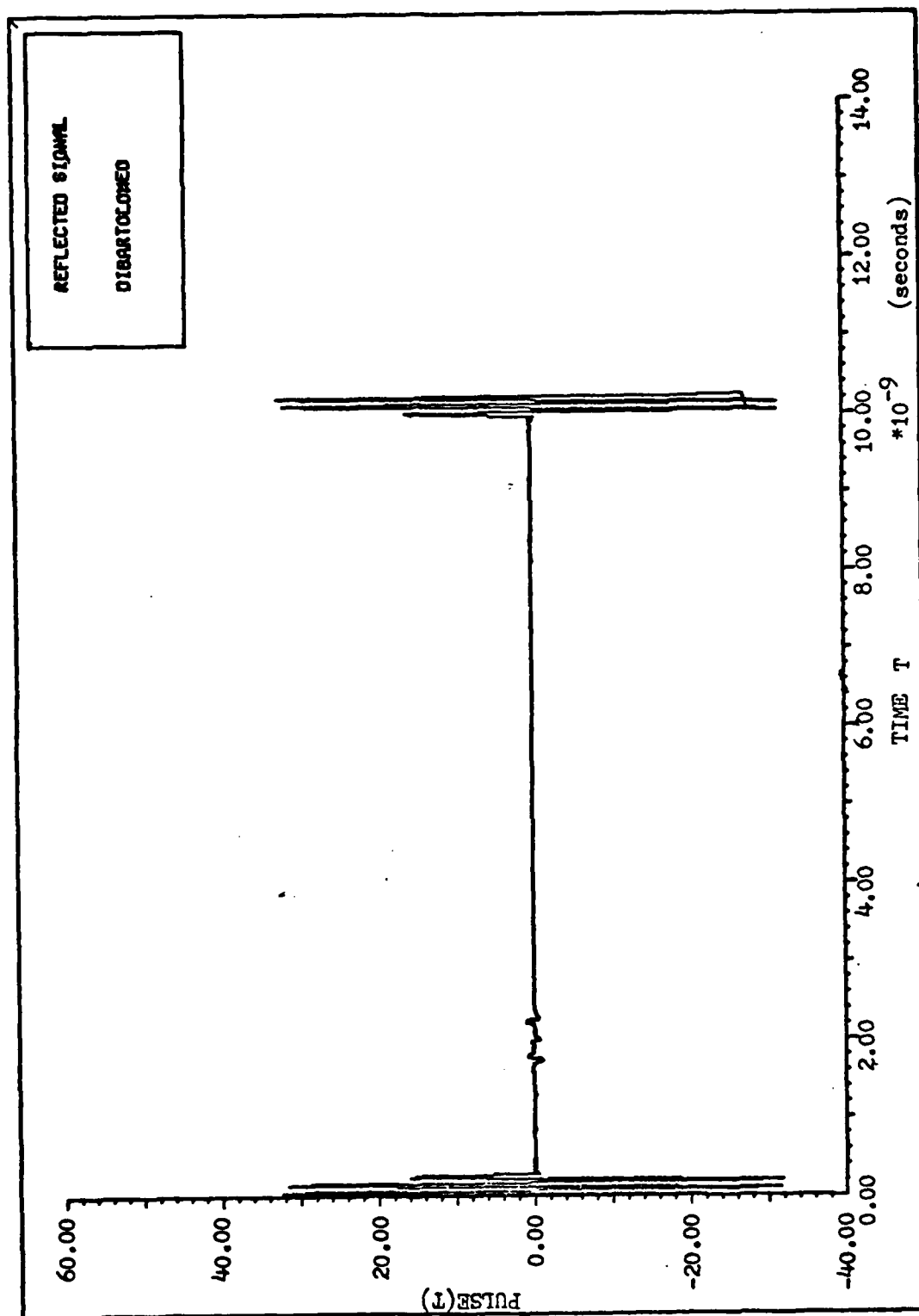


Figure 19. Reflected Signal from Circular Disk with R-F Pulse Incident at  $1^\circ$

which correspond to the five angles are shown in Figures 20-24. The backscattered signals at these angles are shown in Figures 25-29. If more time were available, the backscattered signals would have been inputted to the signal processor outlined in Chapter II, but because of time constraints only the backscattered frequency responses and backscattered signals will be compared. Their similarities and dissimilarities will be pointed out as well as conclusions that can be derived from the figures.

In Figure 20 is shown the frequency response of the backscattering when the incidence direction is  $71^\circ$  off normal. Note the minimum is at approximately 8GHz. As the angle is increased from  $71^\circ$  through  $75^\circ$ , this minimum will move to the right and would pass through 10GHz when the incident angle equals  $73^\circ$  to approximately 13GHz when the angle equals  $75^\circ$ .

If images were able to be made from the backscattered signals corresponding to angle variations around the five angles discussed, one of two things would be true. If the images were virtually the same, then only one of the images would be recorded. Thus, this would reduce the number of image plots to characterize the object. If the image plots were quite dissimilar, then all the image plots obtained would definitely be needed and the angle variations would further have to be subdivided to see if the image plots corresponding to the smaller angle variations were similar or not. This process is repeated until image plots which were virtually the same were obtained. At this point, only the previous image plots

are retained for comparison to the image plot obtained from the unknown object.

In Figures 25 to 29 are shown the backscattered signals when the angle of incidence is  $71^\circ$ ,  $72^\circ$ ,  $73^\circ$ ,  $74^\circ$  and  $75^\circ$  off normal, respectively. Notice that the backscattered signal when  $\theta_0 = 73^\circ$  is the smallest return out of the 5 returns. This makes sense since at  $73^\circ$  the frequency response of the backscattering has a minimum at 10GHz which is the frequency of the incident signal. As  $\theta_0$  is increased and decreased around  $73^\circ$  the backscattered signals are shown to increase in amplitude. This indicates that a valley in the frequency response has been passed through. At the right side of the curves shown in Figures 25-29 there is another response. This is conjectured to be the diffracted return with the dominant curve on the left being the specular return.

#### Backscattering from the Circular Disk at $89^\circ$ off normal.

In Figure 30 is shown the frequency response when  $\theta_0 = 89^\circ$ . This case is considered to illustrate what the return signal looks like at almost edge-on incidence.

In Figures 31 and 32 are shown the backscattered signals at  $\theta_0 = 89^\circ$  for the 10GHz R-F Pulse and video pulse. Note that the amplitudes of the returns are reduced from the amplitudes of the returns centered around  $\theta_0 = 73^\circ$  by 2 orders of magnitude. It can thus be concluded that the diffracted returns are always smaller than the physical optics returns. It is interesting to note the curve on the right side of Figure 32. This return quite possibly could be the pulse traversing the disk and being diffracted back. From



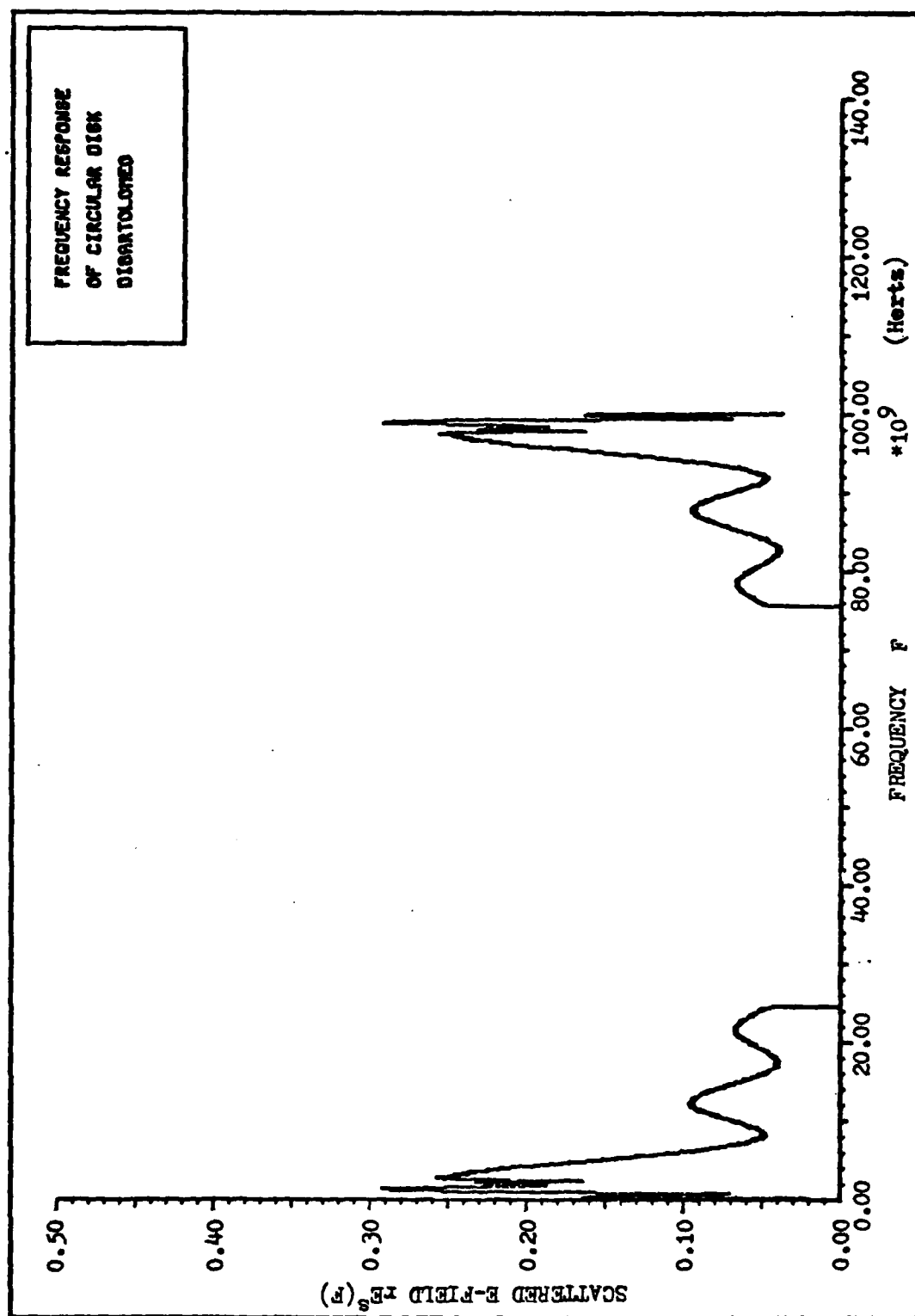


Figure 20. Frequency Response of the Scattering from the Circular Disk with  $\theta_0 = 71^\circ$

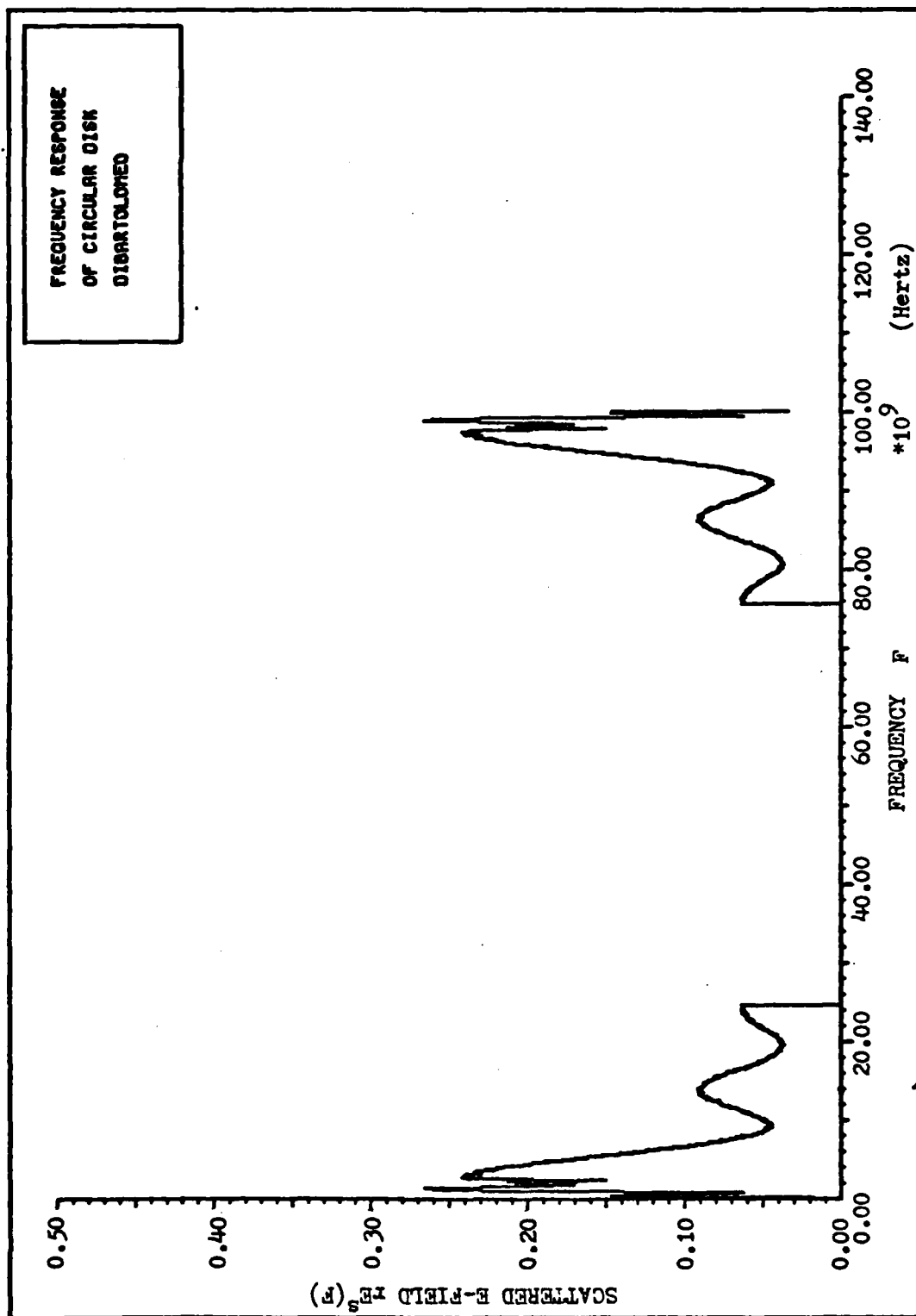


Figure 21. Frequency Response of the Scattering from the Circular Disk with  $\theta_0 = 72^\circ$

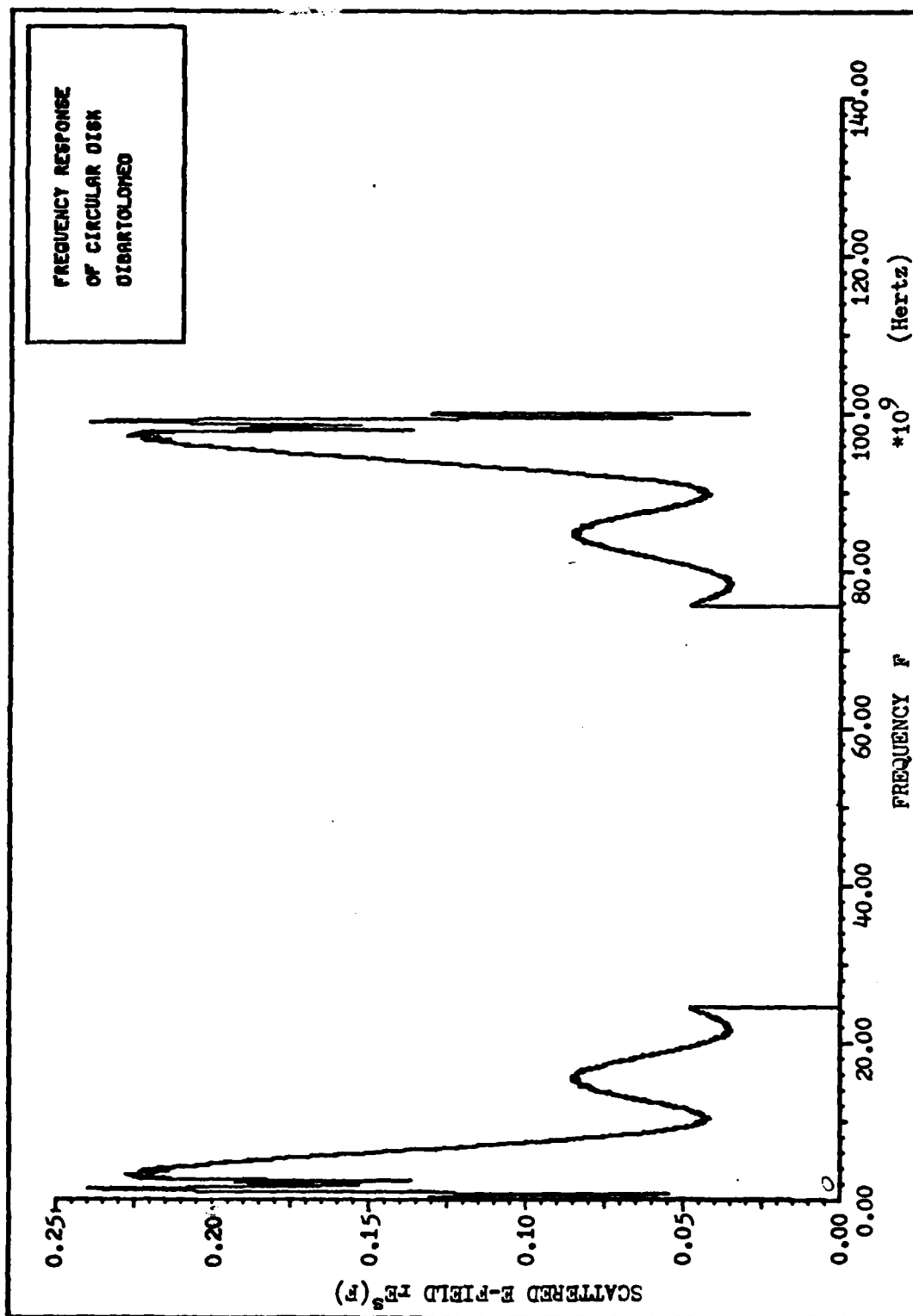


Figure 22. Frequency Response of the Scattering from the Circular Disk with  $\theta_0 = 73^\circ$

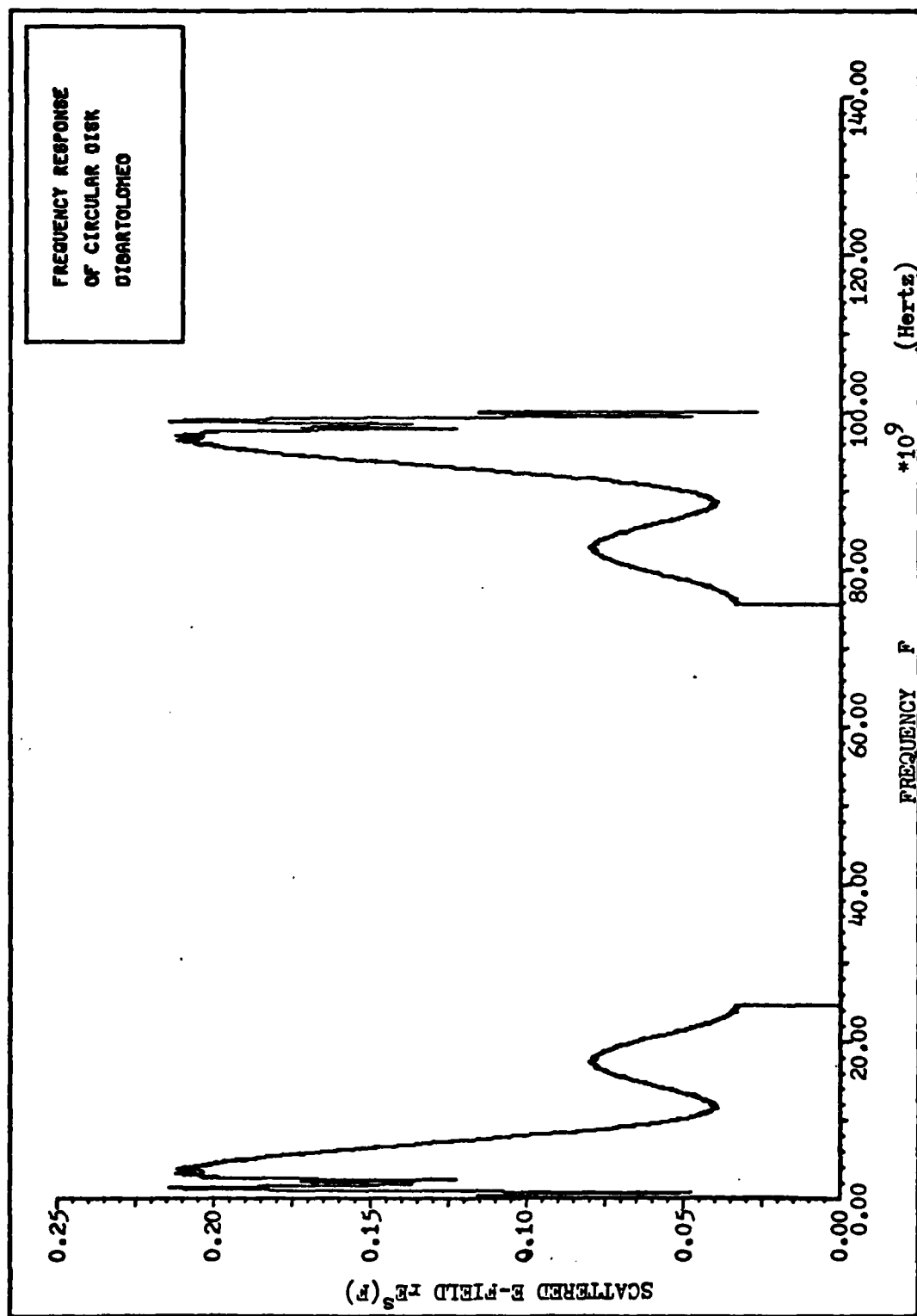


Figure 23. Frequency Response of the Scattering from the Circular Disk with  $\theta_0 = 74^\circ$

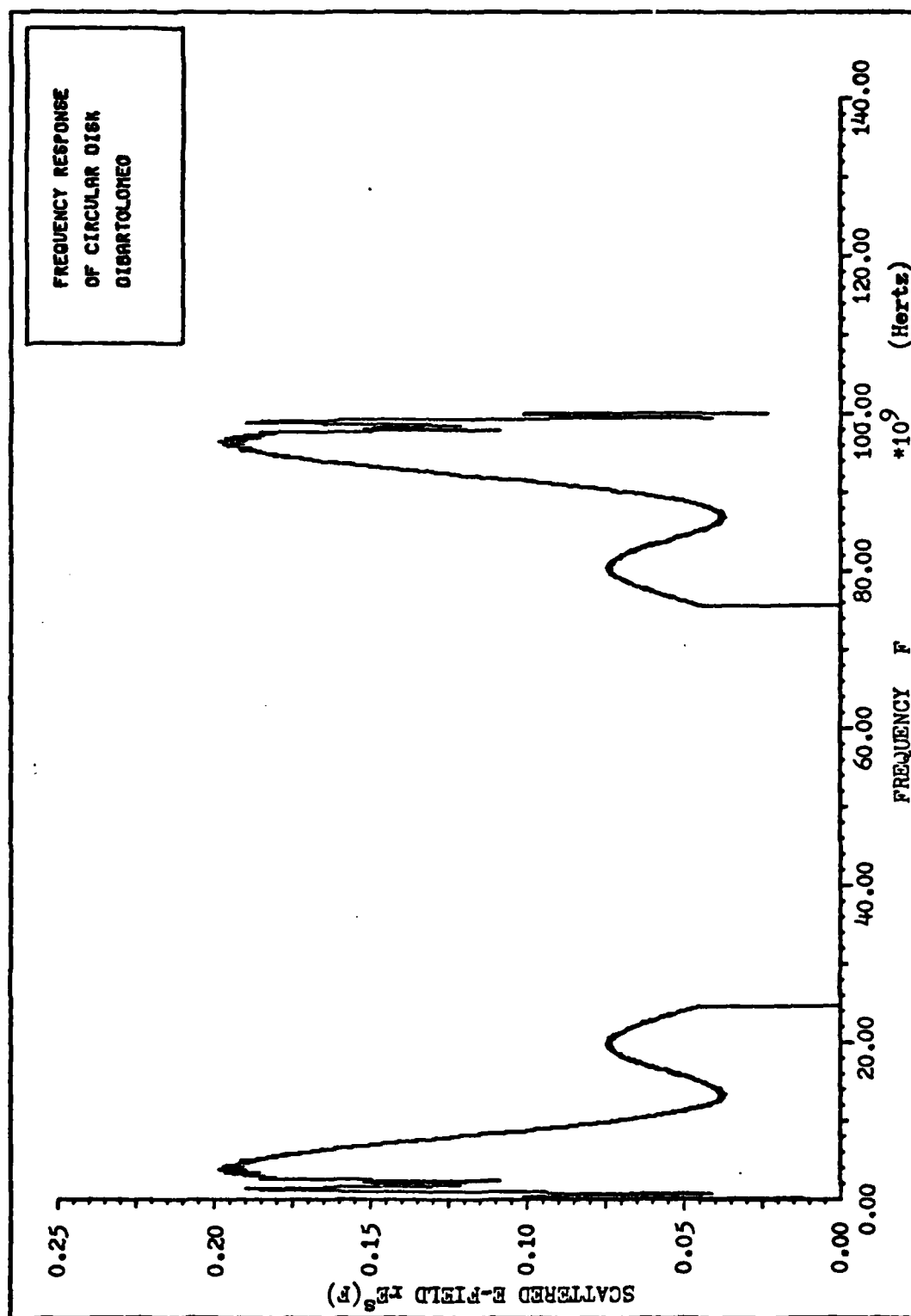


Figure 24. Frequency Response of the Scattering  
from the Circular Disk with  $\theta_0 = 75^\circ$

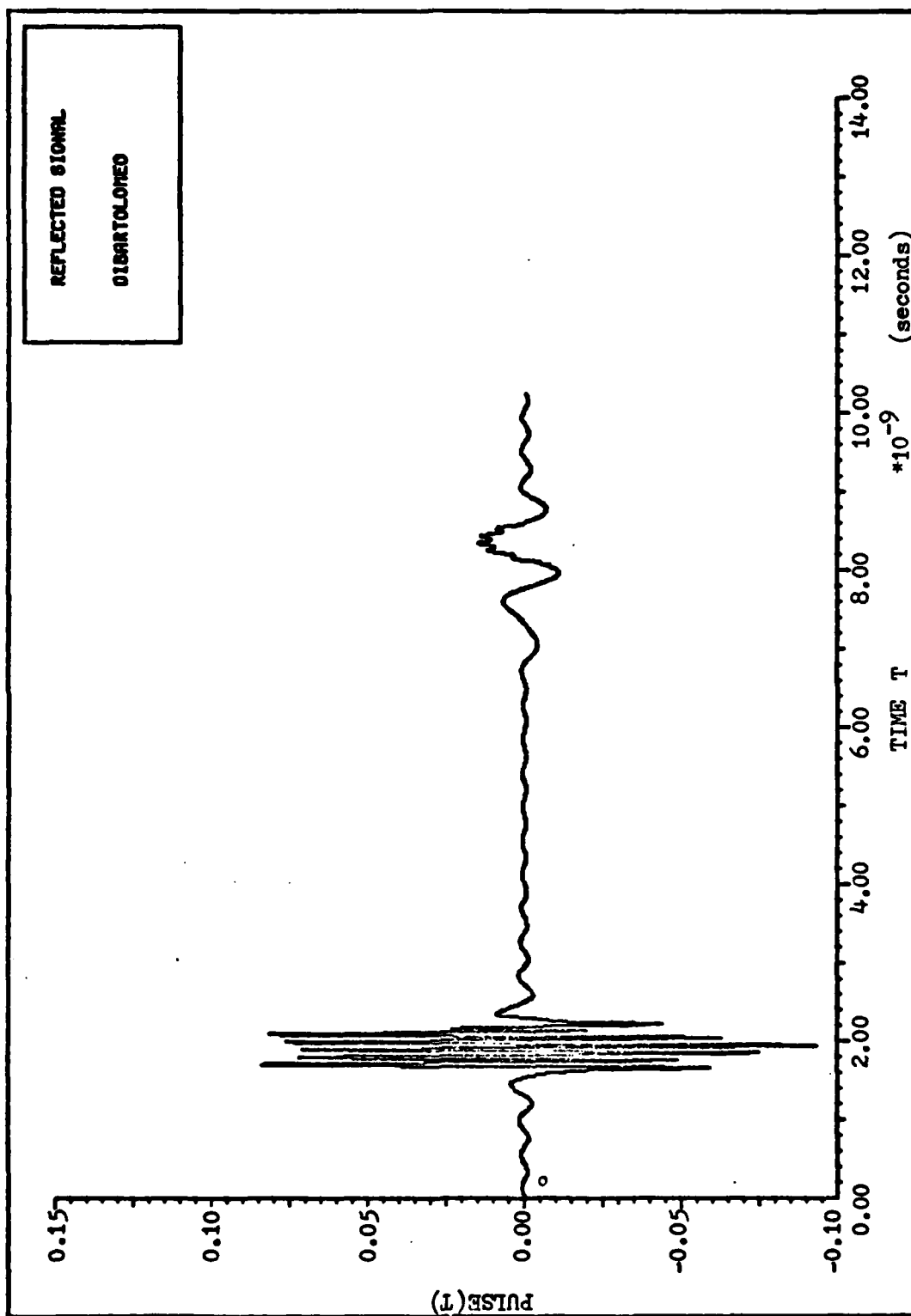


Figure 25. Reflected Signal from the Circular Disk with R-F Pulse Incident at  $71^\circ$

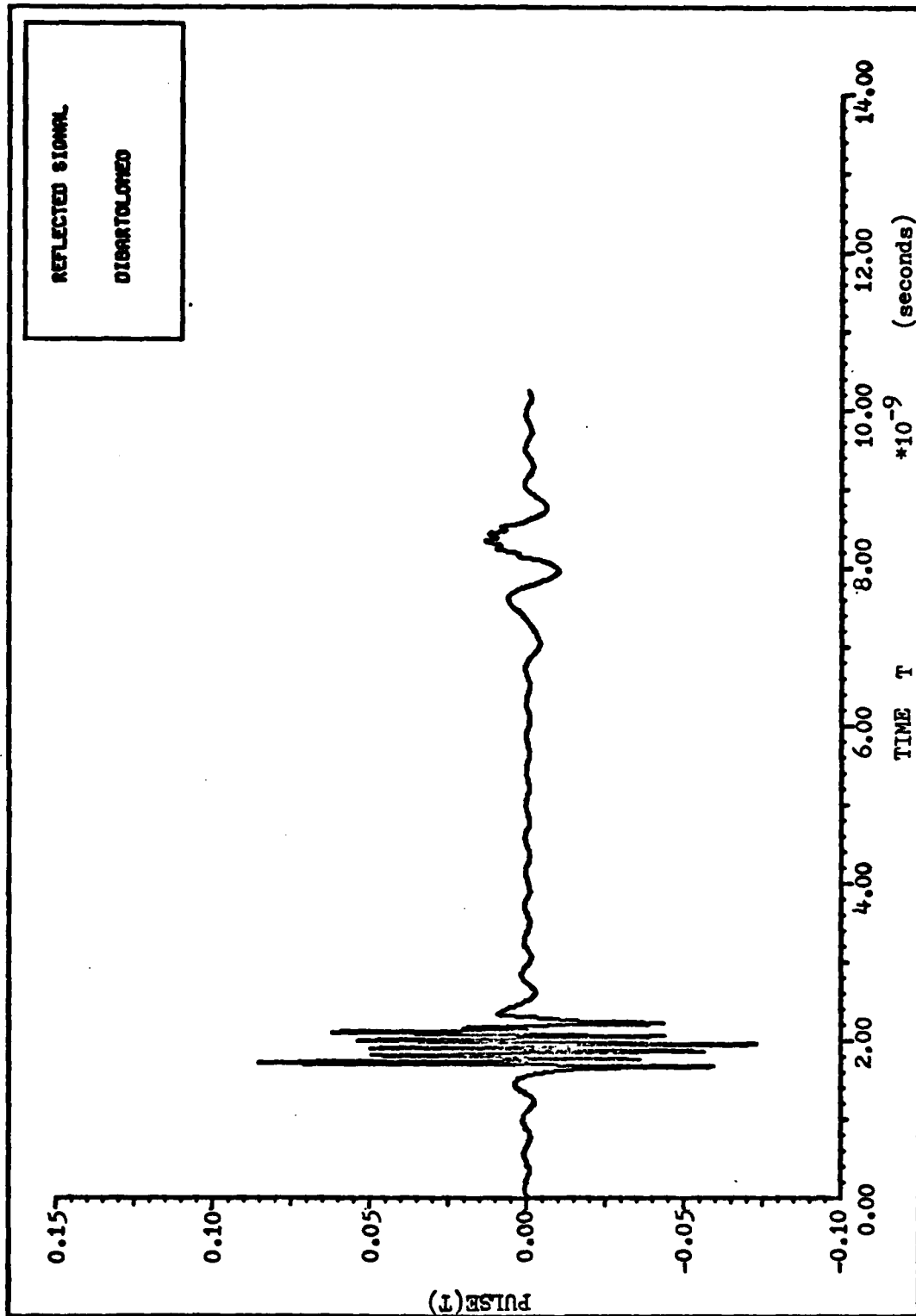


Figure 26. Reflected Signal from the Circular Disk with R-F Pulse Incident at  $72^\circ$

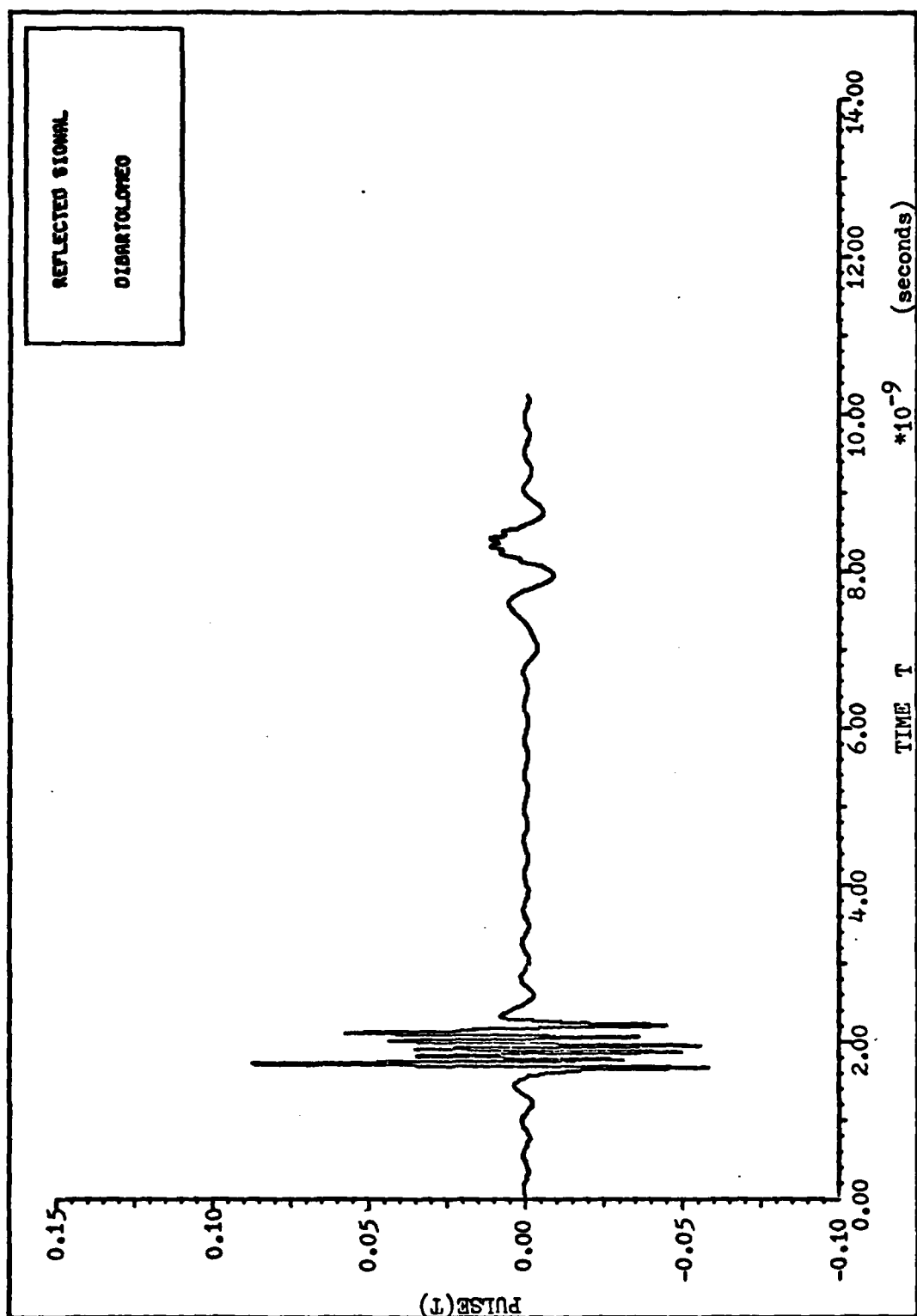


Figure 27. Reflected Signal from the Circular Disk with R-F Pulse Incident at  $73^\circ$



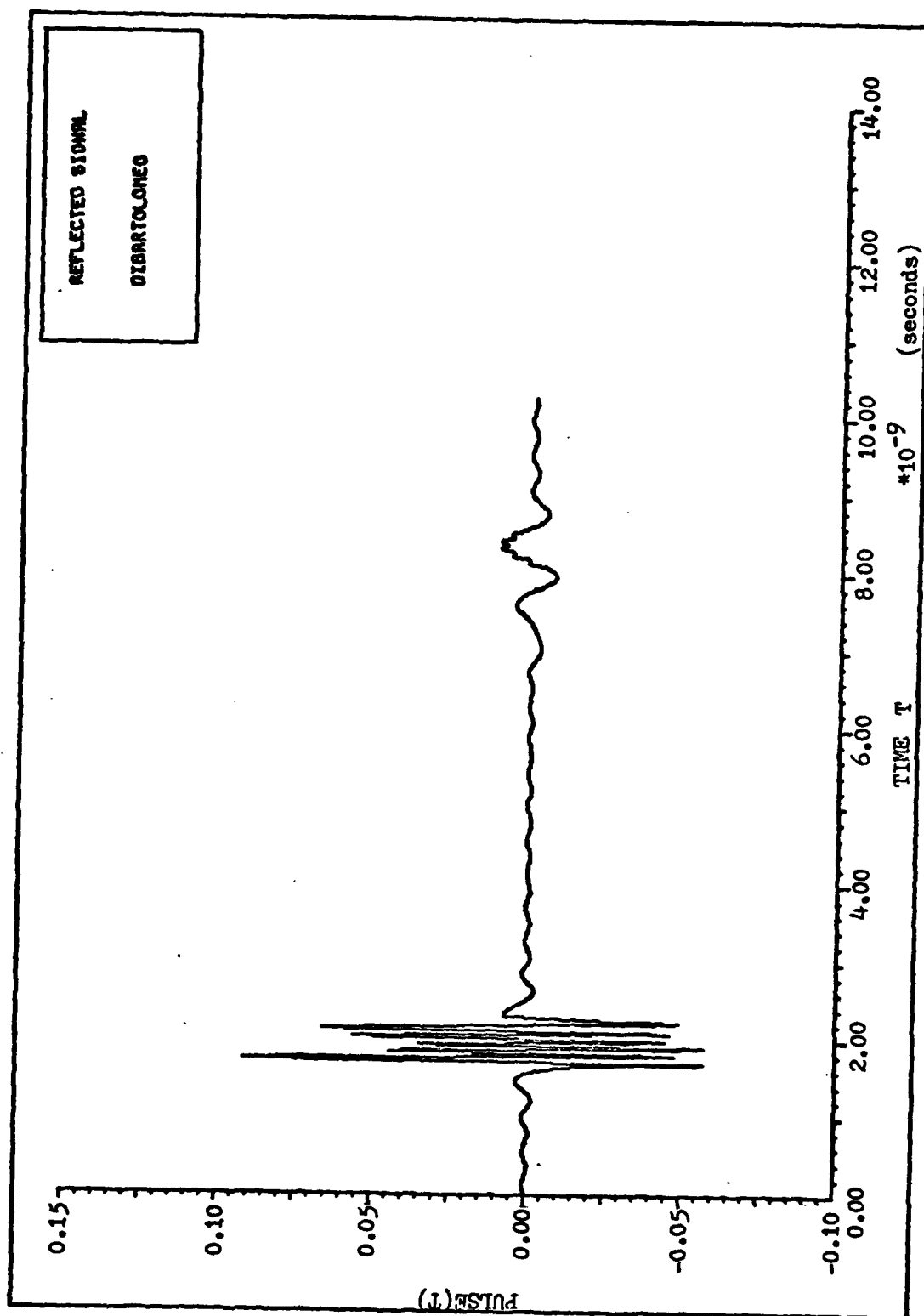


Figure 28. Reflected Signal from the Circular Disk with R-F Pulse Incident at  $74^\circ$

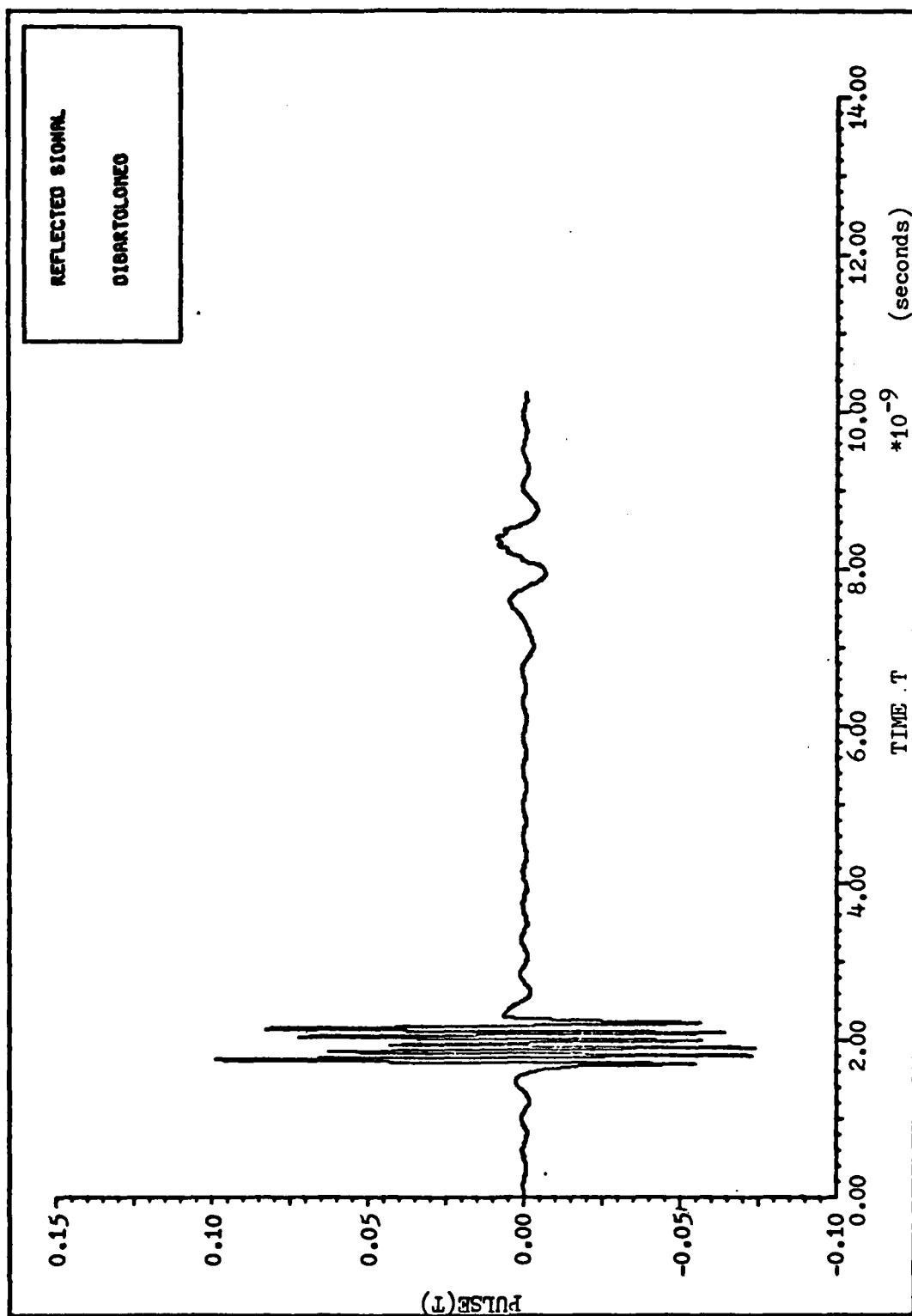


Figure 29. Reflected Signal from the Circular Disk with R-F Pulse Incident at  $75^\circ$

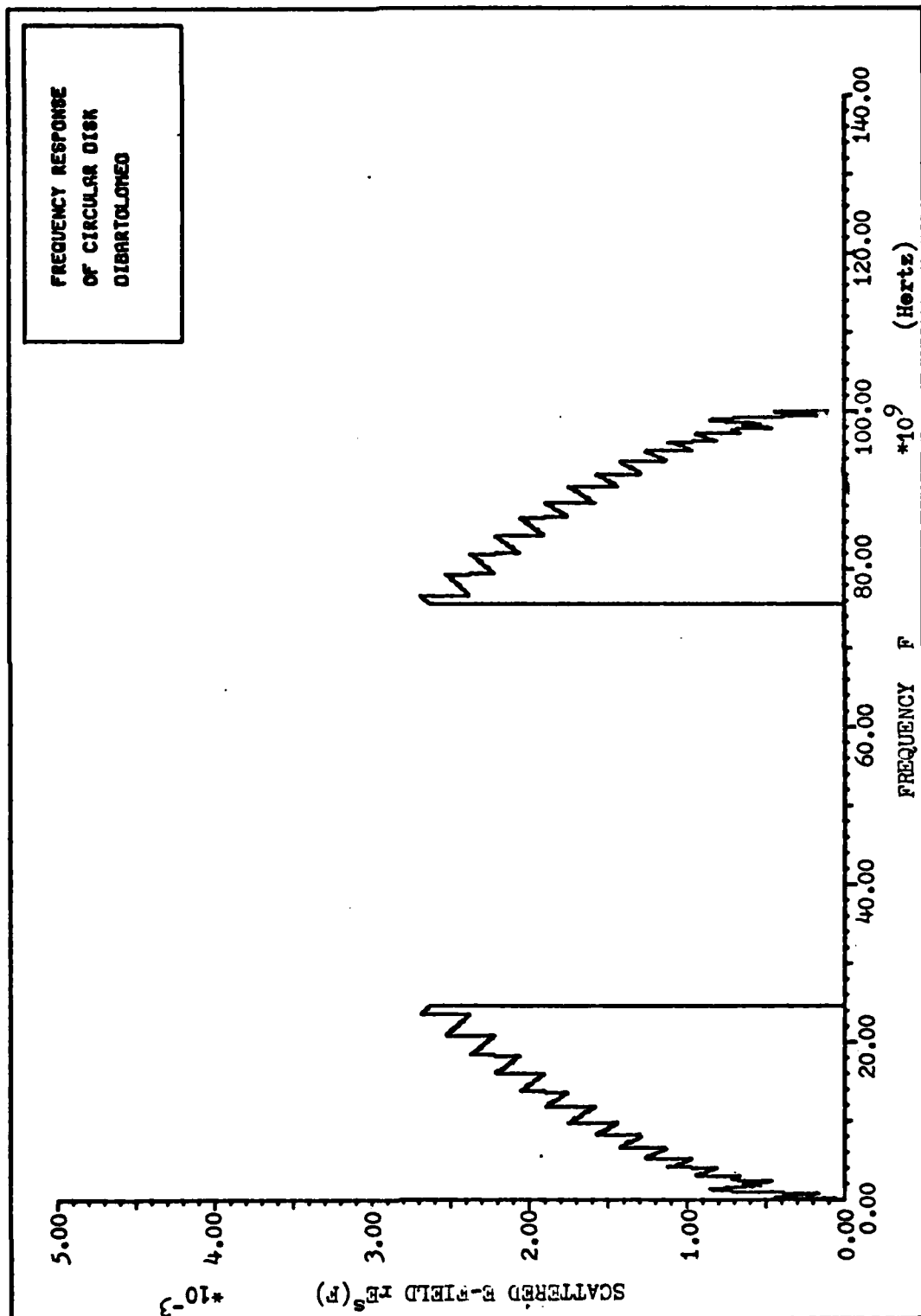


Figure 30. Frequency Response of the Scattering from the Circular Disk with  $\theta_0 = 89^\circ$

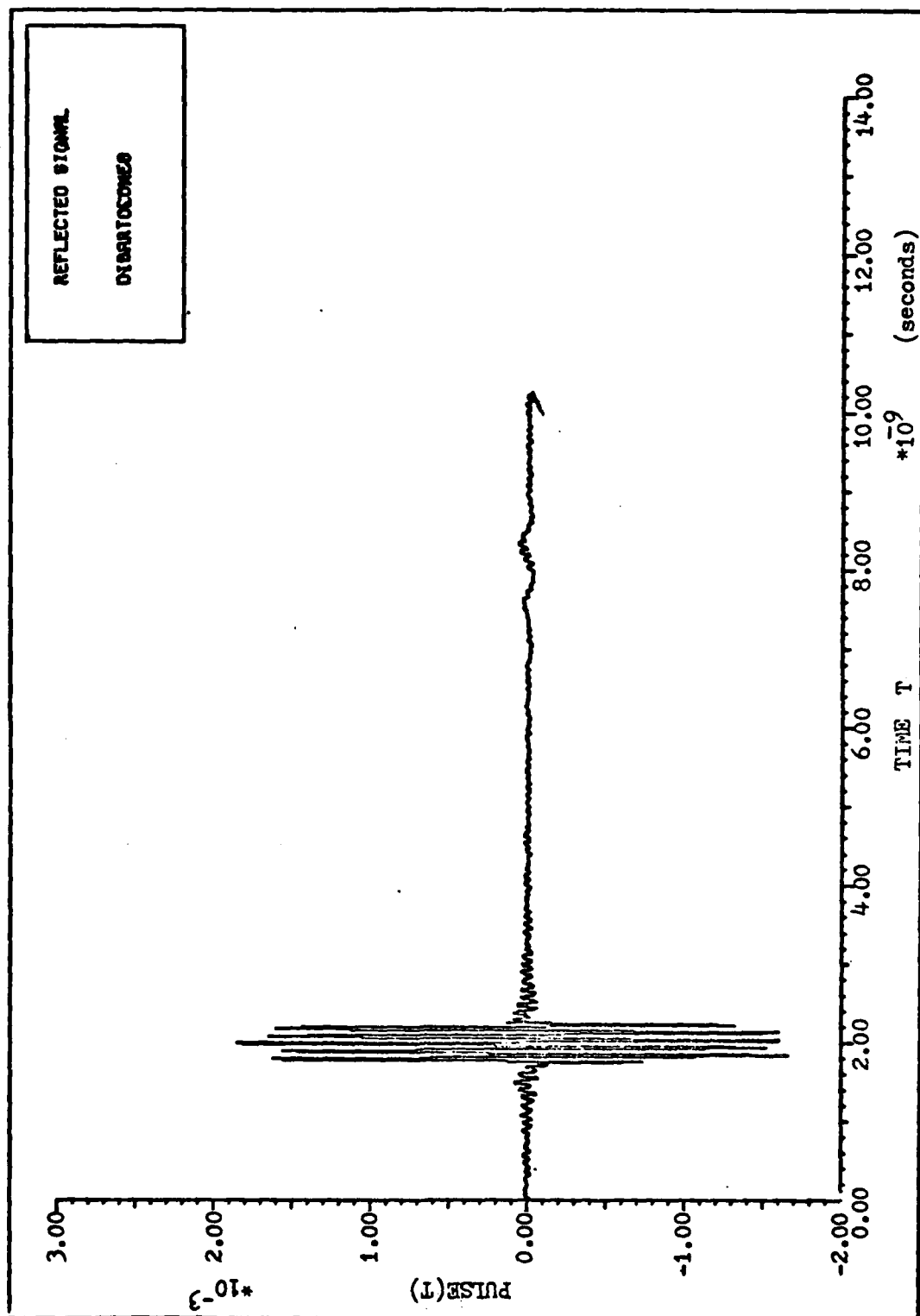


Figure 31. Reflected Signal from the Circular Disk with R-F Pulse Incident at  $89^\circ$

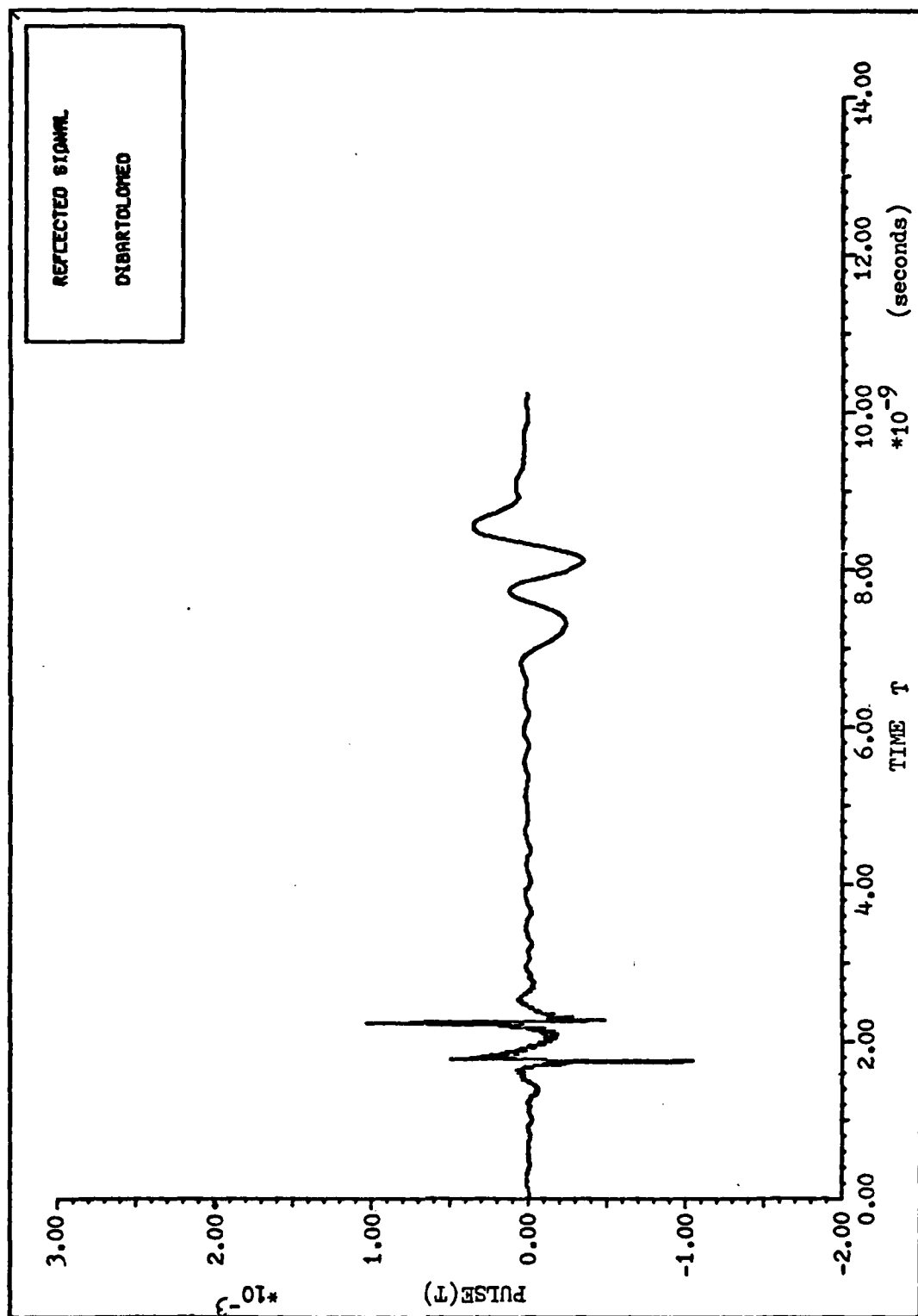


Figure 32. Reflected Signal from the Circular Disk with Video Pulse Incident at  $89^\circ$

Figure 32 it can be observed that the time delay between the beginning of the first return and the beginning of the second return is approximately 5.20 nsec. It can be calculated that the distance in space that corresponds to a time delay of 5.2 nsec is equal to approximately 1.56 m. This means that the incident pulse has to travel an additional 1.56 m to produce the second return than it had to travel to produce the first return. The first return in Figure 32 is considered the physical optics return from the edge of the disk. The second return in Figure 32 is conjectured to be the return caused by the incident pulse traversing the disk and then traveling half way around the disk to the point of incidence. With the radius of the disk equal to 0.30 m, the additional distance that the second return travels is approximately 1.54 m which provides a time delay of approximately 5.20 nsec which corresponds to the time delay between returns in Figure 32.

## VI. Conclusions, Findings, and Recommendations

### Conclusions

There are two major conclusions that can be derived from this thesis.

The first conclusion is that no cross-range information can be obtained from the perfectly-conducting sphere due to the aspect independent nature of the backscattered return. The second conclusion is that, with the 10GHz pluse incident, due to cross-range resolution constraints, only the specular scatterer and not both the specular and creeping wave scatterers will be detected on an image plot. Thus, the image plots obtained from the processing discussed in Chapter II would only show one scatterer and would not aid in the identification of the sphere.

### Findings

The backscattered signals centered around  $\theta_0 = 73^\circ$  for the circular disk are quite similar. The above indicates that maybe the image plots that would be obtained from angle variations around  $71^\circ$ ,  $72^\circ$ ,  $73^\circ$ ,  $74^\circ$  and  $75^\circ$  off normal, quite possibly could be quite similar. If this were the case, the five image plots could be replaced by one image plot. If such were not the case, then these five image plots would definitely be needed in the series of image plots used to characterize the circular disk.

### Recommendations

It is recommended that the processor described in Chapter II be implemented and that the number of distinct image plots that are obtained for the circular disk be investigated. The effect of increasing or decreasing the frequency or, equally, decreasing or increasing the wavelength of the incident E-field, on the cross range resolution in reference to the disk should be investigated. In other words, are the diffraction returns off the edges of the circular disk attenuated as the frequency is increased and is  $\Delta x$ , as defined in Chapter II, too large to resolve the diffraction scatterers as the frequency is decreased.



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## Appendix A

### Explanation and Program Listing for the Frequency Response of the Sphere

The following listing is the computer program that was used to generate Figures' 6, 7, 8, 9, 10, and 11. The program is a straightforward application of the equations for the different frequency regions as outlined in Chapter III. However, a few words will be stated to explain the plotting.

First of all, the plotting was done using a library which is unique to the computer system at the Air Force Institute of Technology. The library of subroutines used is called SAPL. The subroutine within SAPL used to do the plotting is called HGRAPH. As an example, generation of one of the graphs will be explained.

Figure 10 is a graph of the real part of the impulse response of the sphere. The computer program statements which generate this graph are statements 1140 to 1220 inclusive. The array IE which has 17 locations is used for labeling. IE(1) to IE(8) are used to label the upper righthand corner box. IE(9) and IE(10) are used to label the abscissa and IE(11) and IE(12) are used to label the ordinate. IE(13) to IE(17) are used to title the graph (NOTE: 10 hollerith characters fit in one array location). This title goes on the outside of the graph box along the ordinate coordinate. The CALL PLOT statements merely position the pen on the plotter. The CALL HGRAPH statement calls the correct subroutine for plotting. The argument list

contains seven arguments. In the example, V is the array of x-axis values of dimension 1026. ROESR is the array of y-axis values of dimension 1026. Both arrays are two more locations than 1024 for internal scaling purposes. The "1024" is the number of points to be plotted. The IE is the labeling array. The "1" specifies internal scaling by the program. The first "0" means connect successive points by a straight line. And, finally, the second "0" means to draw dots at points as opposed to other symbols (i.e., triangles, circles, etc.).

The preceding is all that is needed to plot graphs using the HGRAPH subroutine in the SAPL library.

```

100=      PROGRAM IMAGE12 (INPUT, OUTPUT, PLOT)
110=      REAL C1, C2, E1, E2, F1, F2, G1, G2, H1, H2, I1, I2, J1, J2, K1, K2,
120=      $ L1, L2, M1, M2, N1, N2, P1, P2, KA
130=      INTEGER Z
140=      COMPLEX C, E, F, G, H, I, J, K, L, M, N, P
150=      $, PULSE (1024), PULSER (1024), RDES (1024)
160=      DIMENSION X (1026), Y (1026), W (1026), V (1026), ID (17)
170=      $, IA (17), IB (17), IC (17), IE (17), RDESR (1026), RDESI (1026)
180=      $, IF (17)
190=C
200=C      FIX DIAMETER OF SPHERE, A, AND DISTANCE TO TARGET, RD.
210=C
220=      A=0.3
230=      RD=0.0
240=      B=0.45
250=C
260=C      COMPUTE WHAT THE SCATTERED FIELD, RDES, AS A FUNCTION
270=C      OF WAVE NUMBER, IS FOR THE LOW FREQUENCY REGION, THE
280=C      LOWER RESONANCE REGION, THE UPPER RESONANCE REGION,
290=C      AND THE HIGH FREQUENCY REGION.
300=C
310=      FREQ=97656000
320=      DO 10 Z=1, 251
330=      KA=((0.3)+(2)+(3.14159)+FREQ+(Z-1))/(300000000)
340=      IF (KA.EQ.0) GO TO 55
350=      C1=0.0
360=      C2=(KA+RD)/(A)
370=      C=CMPLX(C1, C2)
380=      E1=(3./2.)+(KA**3.)+(1.-(5./54.)+(KA**2.)
390=      $ + (17./900.)+(KA**4.)-(6651923./11907000.)+(KA**6.))
400=      E2=(1./2.)+(KA**6.)+(1.+(6./5.)+(KA**2.))
410=      E=CMPLX(E1, E2)
420=      F1=1.0
430=      F2=1./(2.+KA)
440=      F=CMPLX(F1, F2)
450=      G1=0.0
460=      G2=-(2.+KA)
470=      G=CMPLX(G1, G2)
480=      H1=0.0
490=      H2=(3.141593)+(KA-(1./6.))-1.)
500=      H=CMPLX(H1, H2)
510=      I1=1.357588 + (0.741196)+(KA**(-2./3.))
520=      I2=(1.283788)+(KA**(-2./3.))
530=      I=CMPLX(I1, I2)
540=      J1=-(KA**(-1./3.))+(2.200002) +
550=      $ (KA**(-1./3.))+(0.445396)
560=      J2=(KA**(-1./3.))+(1.270172) +
570=      $ (KA**(-1./3.))+(0.257150)
580=      J=CMPLX(J1, J2)
590=      K1=0.695864 + (0.964654)+(KA**(-2./3.))
600=      K2=-(1.670829)+(KA**(-2./3.))
610=      K=CMPLX(K1, K2)
620=      L1=-(KA**(-1./3.))+(7.014224) -
630=      $ (KA**(-1./3.))+(0.444477)
640=      L2=(KA**(-1./3.))+(4.049663) -
650=      $ (KA**(-1./3.))+(0.256619)

```

```

660=      L=CMPLX(L1,L2)
670=      M1=0.807104 + (0.798821)*(KA**(-2./3.))
680=      M2=(1.383598)*(KA**(-2./3.))
690=      M=CMPLX(M1,M2)
700=      N1=-(KA**((1./3.))*(5.048956) -
710=      $ (KA**(-1./3.))*(0.312321)
720=      N2=(KA**((1./3.))*(2.915016) -
730=      $ (KA**(-1./3.))*(0.180319)
740=      N=CMPLX(N1,N2)
750=      P1=0.0
760=      P2=((KA*RD)/(A)) - (2.)*(KA)
770=      P=CMPLX(P1,P2)
780=      IF (KA.LE.0.4) GO TO 20
790=      IF (KA.GT.20.0) GO TO 50
800=      IF (KA.LE.0.8) GO TO 30
810=      IF (KA.GE.0.9) GO TO 40
820= 20    RDES(Z)=(B/KA)*(KA**3.)*(CEXP(C))
830=      GO TO 10
840= 30    RDES(Z)=(((2./3.)*B)/(KA))*(CEXP(C))*(E)
850=      GO TO 10
860= 40    RDES(Z)=((((2./3.)*B)/(KA))*(CEXP(C)))*
870=      $ (-KA/2.)*(CEXP(G))*F + (KA**((4./3.)))*
880=      $ (CEXP(H))*(I)*(CEXP(J)) + (K)*(CEXP(L)) -
890=      $ (M)*(CEXP(N))
900=      GO TO 10
910= 50    RDES(Z)=(-B/3.)*(CEXP(P))
920=      GO TO 10
930= 55    RDES(Z)=(0.,0.)
940= 10    CONTINUE
950=      DO 60 Z=1,251
960=      A=REAL(RDES(Z))
970=      B=AIMAG(RDES(Z))
980=      RDES(Z)=CMPLX(A,-B)
990=      X(Z)=REAL(RDES(Z))
1000=     Y(Z)=AIMAG(RDES(Z))
1010= 60    W(Z)=(((X(Z)**2.)+(Y(Z)**2.))**((1./2.))
1020=     DO 65 Z=252,774
1030=     RDES(Z)=(0.,0.)
1040= 65    W(Z)=0.0
1050=     DO 68 Z=1,250
1060=     RDES(1025-Z)=CMPLX(X(Z+1),-(Y(Z+1)))
1070= 68    CONTINUE
1080=     CALL FFT(RDES,10,-1.)
1090=     DO 1 Z=1,1024
1100=     RDESR(Z)=REAL(RDES(Z))
1110=     RDESI(Z)=AIMAG(RDES(Z))
1120=     V(Z)=(Z-1.)/(10**11.)
1130= 1     CONTINUE
1140=     DATA IE(1)/20H REAL PART OF
1150=     DATA IE(3)/20H IMPULSE RESPONSE
1160=     DATA IE(5)/20H OF SPHERE
1170=     DATA IE(7)/20H DIBARTOLOMED
1180=     DATA IE(9)/20H T
1190=     DATA IE(11)/20H IMPULSE RESPONSE
1200=     DATA IE(13)/50H

```

```

1210=      CALL PLOT(0.,-4.,-3.) $ CALL PLOT(0.,.03,-3.)
1220=      CALL HGRAPH(V,RDESP,1024,IE,1,0,0)
1230=      DATA IF(1)/20H IMAGINARY PART OF /
1240=      DATA IF(3)/20H IMPULSE RESPONSE /
1250=      DATA IF(5)/20H OF SPHERE /
1260=      DATA IF(7)/20H DIAPARTOLOMED /
1270=      DATA IF(9)/20H T /
1280=      DATA IF(11)/20H IMPULSE RESPONSE /
1290=      DATA IF(13)/50H

/
1300=      CALL PLOT(0.,-4.,-3.) $ CALL PLOT(0.,.03,-3.)
1310=      CALL HGRAPH(V,RDESI,1024,IF,1,0,0)
1320=      CALL FFT(RDES,10,1.)
1330=      DO 69 Z=775,1024
1340=      X(Z)=REAL(RDES(Z))
1350=      Y(Z)=AIMAG(RDES(Z))
1360=      W(Z)=(((X(Z))**2.)+(Y(Z)**2.))**(.5/2.))
1370= 69    CONTINUE
1380=      DO 70 Z=1,20
1390=      PRINT*,X(Z),Y(Z),W(Z)
1400= 70    CONTINUE
1410=      DO 80 Z=1,1024
1420=      V(Z)=FREQ*Z
1430= 80    CONTINUE
1440=      DATA ID(1)/20H FREQUENCY RESPONSE /
1450=      DATA ID(3)/20H OF SPHERE /
1460=      DATA ID(5)/20H DIAPARTOLOMED /
1470=      DATA ID(7)/20H /
1480=      DATA ID(9)/20H F /
1490=      DATA ID(11)/20H RDES(F) /
1500=      DATA ID(13)/50H FREQUENCY RESPONSE OF SPHERE

/
1510=      CALL PLOT(0.,-4.,-3.) $ CALL PLOT(0.,.03,-3.)
1520=      CALL HGRAPH(V,W,1024,ID,1,0,0)
1530=      DO 90 Z=1,25
1540=      X=(Z+25)/(10**11.)
1550=      PULSE1=SIN(2*3.14159*(10**10.)*X)
1560=      PULSE2=0.0
1570=      PULSE(Z)=CMPLX(PULSE1,PULSE2)
1580= 90    CONTINUE
1590=      DO 100 Z=26,999
1600=      PULSE1=0.0
1610=      PULSE2=0.0
1620=      PULSE(Z)=CMPLX(PULSE1,PULSE2)
1630= 100   CONTINUE
1640=      DO 105 Z=1000,1024
1650=      X=(Z-999)/(10**11.)
1660=      PULSE1=SIN(2*3.14159*(10**10.)*X)
1670=      PULSE2=0.0
1680=      PULSE(Z)=CMPLX(PULSE1,PULSE2)
1690= 105   CONTINUE
1700=      DO 110 Z=1,1024
1710=      X(Z)=REAL(PULSE(Z))
1720=      V(Z)=(Z-1.)/(10**11.)
1730= 110   CONTINUE
1740=      DATA IA(1)/20H INCIDENT /
1750=      DATA IA(3)/20H PULSE /

```

```

1760= DATA IA(5)/20H DIBARTOLOMED /
1770= DATA IA(7)/20H /
1780= DATA IA(9)/20H T /
1790= DATA IA(11)/20H PULSE(T) /
1800= DATA IA(13)/50H INCIDENT PULSE OF RF AT 10 GHZ

1810= CALL PLOT(0.,-4.,-3.) $ CALL PLOT(0.,.03,-3.)
1820= CALL HGRAPH(V,X,1024,IA,1,0,0)
1830= CALL PLOT(0.,-4.,-3.) $ CALL PLOT(0.,.03,-3.)
1840= CALL HGRAPH(V,X,200,IA,1,0,0)
1850= CALL FFT(PULSE,10,1.)
1860= DO 120 Z=1,1024
1870= X(Z)=REAL(PULSE(Z))
1880= Y(Z)=AIMAG(PULSE(Z))
1890= W(Z)=((((X(Z))**2.)+(Y(Z)**2.))**.5)**(1./2.)
1900= 120 V(Z)=FREQ*Z
1910= DATA IB(1)/20H TRANSFORM OF /
1920= DATA IB(3)/20H PULSE /
1930= DATA IB(5)/20H DIBARTOLOMED /
1940= DATA IB(7)/20H /
1950= DATA IB(9)/20H F /
1960= DATA IB(11)/20H PULSE(F) /
1970= DATA IB(13)/50H FOURIER TRANSFORM OF PULSE

1980= CALL PLOT(0.,-4.,-3.) $ CALL PLOT(0.,.03,-3.)
1990= CALL HGRAPH(V,W,200,IB,1,0,0)
2000= DO 140 Z=1,1024
2010= PULSER(Z)=(ROES(Z))*PULSE(Z)
2020= 140 CONTINUE
2030= CALL FFT(PULSER,10,-1.)
2040= DO 150 Z=1,1024
2050= X(Z)=REAL(PULSER(Z))
2060= Y(Z)=AIMAG(PULSER(Z))
2070= W(Z)=((((X(Z))**2.)+(Y(Z)**2.))**.5)**(1./2.)
2080= 150 V(Z)=(Z-1.)/(10**11.)
2090= CONTINUE
2100= DATA IC(1)/20H REFLECTED /
2110= DATA IC(3)/20H SIGNAL /
2120= DATA IC(5)/20H DIBARTOLOMED /
2130= DATA IC(7)/20H /
2140= DATA IC(9)/20H T /
2150= DATA IC(11)/20H PULSER(T) /
2160= DATA IC(13)/50H REFLECTED PULSE OF RF

2170= CALL PLOT(0.,-4.,-3.) $ CALL PLOT(0.,.03,-3.)
2180= CALL HGRAPH(V,X,1024,IC,1,0,0)
2190= CALL PLOT(0.,-4.,-3.) $ CALL PLOT(0.,.03,-3.)
2200= CALL HGRAPH(V,Y,1024,IC,1,0,0)
2210= CALL FFT(PULSE,10,-1.)
2220= DO 160 Z=1,1024
2230= X(Z)=REAL(PULSE(Z))
2240= 160 V(Z)=(Z-1.)/(10**11.)
2250= CALL PLOT(0.,-4.,-3.) $ CALL PLOT(0.,.03,-3.)
2260= CALL HGRAPH(V,X,1024,IA,1,0,0)
2270= CALL PLOT(M)
2280= STOP
2290= END
2300= SUBROUTINE FFT(X,M,XI)

```



```

2310=    COMPLEX X(1),U,W,T
2320=    N=2**M
2330=    NV2=N/2
2340=    NM1=N-1
2350=    J=1
2360=    DO 7 I=1,NM1
2370=    IF(1.GE.J) GO TO 5
2380=    T=X(J)
2390=    X(J)=X(I)
2400=    X(I)=T
2410= 5   K=NV2
2420= 6   IF(K.GE.J) GO TO 7
2430=    J=J-K
2440=    K=K/2
2450=    GO TO 6
2460= 7   J=J+K
2470=    PI=3.14159265358979
2480=    DO 20 L=1,M
2490=    LE=2**L
2500=    LE1=LE/2
2510=    U=(1.0,0.0)
2520=    W=CEXP(CMPLX(0.,-XI*PI/LE1))
2530=    DO 20 J=1,LE1
2540=    DO 10 I=J,N,LE
2550=    IP=I+LE1
2560=    T=X(IP)*U
2570=    X(IP)=X(I)-T
2580= 10  X(I)=X(I)+T
2590= 20  U=U*W
2600=    IF(X1.GT.0.) RETURN
2610=    DO 30 I=1,N
2620= 30  X(I)=X(I)/N
2630=    RETURN
2640=    END

```

## Appendix B

### Explanation and Program Listings for the Frequency Response of the Far Field Scattering of the Perfectly-Conducting Infinitely-Thin, Circular Disk

The first computer program shown on the following pages calculates the far field radar cross section and associated E-field for the infinitely thin circular disk (Ref 9). A sample of the program I/O is shown after the program listing. The user need only input eight variables for the initial case to be executed. They are:

1.  $KA$  = electrical radius of the disk =  $(2\pi/\lambda)a$ .
2. THETA INCIDENT =  $\theta_0$  (degrees)
3. POLARIZATION =  $\alpha$  (degrees)
4. THETA SCATTERED =  $\theta_s$  (degrees)
5. PHI SCATTERED =  $\phi_s$  (degrees)

WHICH VARIABLE IS TO BE INCREMENTED? :

if 1 through 5: the variable associated with that index  
above will be incremented

if 1 through 5: then only the initial case will be  
calculated. In this event, the remaining  
parameters are not requested.

TYPE NUMBER OF CASES: enter the total number of computa-  
tions to be performed.

WHAT IS THE INCREMENT?: enter the increment by which the variable selected should be increased after each execution.

All inputs may be made in free format. The program labels the first column with the name of the variable to be incremented. The second and third columns contain the cross sections associated with the  $\theta$  and  $\phi$  components of the scattered field. The last four columns list the magnitudes and phases (in degrees) of both the  $\theta$  and  $\phi$  components of the normalized scattered electric field.

On completion of the cases desired, the program will ask for a new KA and proceed as before. If a 0 is entered, the program will terminate. If a -1 is entered, a brief statement of the problem geometry will be given. Following this statement, the program will ask for a new value of KA.

The program works fine up to  $KA=15$ . For  $KA>15$ , a high frequency approximation is used as is outlined in Chapter IV. The program to implement the high frequency approximation is called SCATTER and follows the I/O of the first program. The output for  $\theta_0=45^\circ$  follows SCATTER's program listing. It is important to note that in relation to the input parameters of the first program, SCATTER is restrictive in the sense that it returns only the theta components of cross section and E-field and assumes the phi components of cross section and E-field are zero ( $\alpha=0$ ). Also the incident angle is equal to the scattered angle (backscattering) and is

restricted between  $0^\circ$  and  $90^\circ$ . It is also restrictive because it will only increment KA.

Although the first program's output and the output from SCATTER have values for the cross section and E-field from KA=1-25, the first program is invalid for KA>15 and SCATTER is invalid for the lower values of KA. What is needed is a program that combines the two. This combination program follows the output from SCATTER. The output from the combination program follows its listing. Note that for KA $\leq$ 15, it uses values from the first program and for KA>15, it uses values from SCATTER.

In the SCATTER program, the Fresnel Integrals were calculated using the trapezoidal rule for approximation. The values obtained from the FRESNEL subroutine agree with the literature out to three decimal places (Ref 11:34).

```

100=      PROGRAM DISK24 (INPUT, OUTPUT, TAPE5=INPUT, TAPE6=OUTPUT)
110=      FAP FIELD SCATTERING BY A CIRCULAR METALLIC DISK
120=      DIMENSION LABEL (5), VAR (5)
130=      COMPLEX F4, PSI, W, YD, YETAD, U, X, Z, ZA, ZB, ZC, ZD
140=      1, IX, EPART, EPAPP, EPERT, EPEPP, ESNT, ESNP
150=      COMMON EIG (50), D (50, 50), DNEG (50), R1 (50), F4 (50)
160=      1, SD (50), P (50), SETA (50), SETAD (50), PSI (50), W (50), YD (50)
170=      1, YETAD (50), U (50), X (50), CMPHI (50), SMPHI (50)
180=      DATA LABEL (1)/10H      KA      /
190=      DATA LABEL (2)/10H THETA I  /
200=      DATA LABEL (3)/10H      POL  /
210=      DATA LABEL (4)/10H THETA S  /
220=      DATA LABEL (5)/10H      PHI S  /
230=      IX=(0., 1.)
240=      WRITE (6, 11)
250= 11    FORMAT (1X, ///, 1X, "SCATTERING BY A METALLIC CIRCULAR DISK",
260=      1/, 5X, " (HODGE -- VERSION 12/17/78) ")
270=      WRITE (6, 26)
280= 26    FORMAT (1X, ///, 1X, " (TYPE KA=0 TO STOP PROGRAM) ", /,
290=      11X, " (TYPE KA=-1 FOR A DESCRIPTION OF THE",
300=      1" PARAMETERS) ", /, 1X, " (NORMALIZATION:  ESCAT=A+EINC+ENDRM"
310=      1"/(2+R)+EXP(-J+K+R)) ", /, 1X, " (ALL ANGLES IN DEGREES) ")
320= 4      NDC=1
330=      INDEX=1
340=      WRITE (6, 27)
350= 27    FORMAT (1X, ///, 1X, "1.  KA", 14X, "= ")
360=      READ (5, *) VAR (1)
370=      IF (VAR (1).EQ.-1) GO TO 41
380=      IF (VAR (1).LE.0.) GO TO 5
390=      WRITE (6, 28)
400= 28    FORMAT (1X, "2.  THETA INCIDENT  = ")
410=      READ (5, *) VAR (2)
420=      WRITE (6, 29)
430= 29    FORMAT (1X, "3.  POLARIZATION    = ")
440=      READ (5, *) VAR (3)
450=      WRITE (6, 30)
460= 30    FORMAT (1X, "4.  THETA SCATTERED = ")
470=      READ (5, *) VAR (4)
480=      WRITE (6, 31)
490= 31    FORMAT (1X, "5.  PHI SCATTERED   = ")
500=      READ (5, *) VAR (5)
510=      WRITE (6, 3)
520= 3      FORMAT (1X, //, 1X, "WHICH VARIABLE IS TO BE INCREMENTED?")
530=      READ (5, *) NVAR
540=      IF ((NVAR.LE.0).OR.(NVAR.GT.5)) GO TO 23
550=      WRITE (6, 21)
560= 21    FORMAT (1X, "TYPE NUMBER OF CASES:")
570=      READ (5, *) NDC
580=      WRITE (6, 32)
590= 32    FORMAT (1X, "WHAT IS THE INCREMENT?")
600=      READ (5, *) VINCRE
610= 23    CONTINUE
620=      WRITE (6, 24) LABEL (NVAR)
630= 24    FORMAT (1X, //, A10, 4X, "CROSS SECTION", 22X, "E NORM", /,
640=      113X, "SIGMA/(PI+A**2)", 11X, "THETA", 15X, "PHI", /, 13X,
650=      1"THETA", 7X, "PHI", 8X, "MAG", 5X, "PHASE", 6X, "MAG", 5X,

```

```

660=      1"PHASE",/
670= 6      C=VAR(1)
680=      MMMAX=45
690=      NNMAX=45
700=      IRRMAX=45
710=      THED=VAR(2)*3.14159/180
720=      ETAD=COS(THED)
730=      THE=VAR(4)*3.14159/180
740=      ETA=COS(THE)
750=      PHI=VAR(5)*3.14159/180
760=      ALF=VAR(3)*3.14159/180
770=      CALF=COS(ALF)
780=      SALF=SIN(ALF)
790= 34      DO 10 MM=1,MMMAX
800=          M=MM-1
810=          IQ=0
820=          CALL SMND(M,NNMAX)
830=          CALL DBEIGN(C,M,NNMAX)
840=          CALL DBCOFN(C,M,MMMAX,NNMAX,IRRMAX)
850=          CALL DNEGN(C,M,NNMAX)
860=          CALL DBRAD(C,M,IQ,MMMAX,NNMAX,IRRMAX)
870=          IF(IQ.EQ.1)GO TO 10
880=          CALL FPSI(M,NNMAX)
890=          CALL POLYN(ETAD,M,IRRMAX)
900=          CALL DBANG(NNMAX,IRRMAX)
910=          DO 7 I=1,NNMAX
920= 7          SETA(I)=SETA(I)
930=          CALL FW(M,NNMAX)
940=          CALL POLYN(ETA,M,IRRMAX)
950=          CALL DBANG(NNMAX,IRRMAX)
960=          CALL FY(M,NNMAX)
970= 10          CONTINUE
980=          DO 13 MM=1,MMMAX
990=          CALL FXU(MM,MMMAX)
1000=          IF(MMMAX.LT.MM)GO TO 13
1010=          CALL CSPHI(MM,PHI)
1020= 13          CONTINUE
1030=          CALL FZ(MMMAX,Z,ZA,ZB,ZC,ZD)
1040=          EPART=ETA*(-2*Z*CMPHI(2)+ZA)
1050=          EPARP=-2*Z*SMPHI(2)+ZB
1060=          EPERT=ETA*(2*Z*SMPHI(2)-ZC)
1070=          EPERP=-2*Z*CMPHI(2)+ZD
1080=          IF(ETAD.EQ.0.)GO TO 2
1090=          ESNT=2*IX*(CALF*EPART/ETAD+SALF*EPERT)/C
1100=          ESNP=2*IX*(CALF*EPARP/ETAD+SALF*EPERP)/C
1110=          GO TO 8
1120= 2          ESNT=2*IX*SALF*EPERT/C
1130=          ESNP=2*IX*SALF*EPERP/C
1140= 8          EMAGT=CABS(ESNT)
1150=          EMAGP=CABS(ESNP)
1160=          SIGTHE=EMAGT*EMAGT
1170=          SIGPHI=EMAGP*EMAGP
1180=          E1=REAL(ESNT)
1190=          E2=AIMAG(ESNT)
1200=          IF(E1.EQ.0.)GO TO 16

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1210=      ARG=E2/E1
1220=      EPHAT=180/3.14159*ATAN(ARG)
1230=      IF((E1.LT.0.).AND.(E2.GT.0.)) EPHAT=EPHAT+180
1240=      IF((E1.LT.0.).AND.(E2.LT.0.)) EPHAT=EPHAT-180
1250=      GO TO 17
1260= 16    EPHAT=90
1270=      IF(E2.LT.0.) EPHAT=-90
1280= 17    E1=REAL(ESNP)
1290=      E2=AIMAG(ESNP)
1300=      IF(E1.EQ.0.) GO TO 18
1310=      ARG=E2/E1
1320=      EPHAP=180/3.14159*ATAN(ARG)
1330=      IF((E1.LT.0.).AND.(E2.GT.0.)) EPHAP=EPHAP+180
1340=      IF((E1.LT.0.).AND.(E2.LT.0.)) EPHAP=EPHAP-180
1350=      GO TO 19
1360= 18    EPHAP=90
1370=      IF(E2.LT.0.) EPHAP=-90
1380= 19    IF(NVAR.EQ.0) NVAR=1
1390=      WRITE(6,25) VAR(NVAR),SIGTHE,SIGPHI,EMAGT,EPHAT,
1400=      1EMAGP,EPHAP
1410= 25    FORMAT(1X,F7.2,2(1X,E10.3),2(1X,E10.3,1X,F7.2))
1420= 37    IF(INDEX.EQ.NDC) GO TO 4
1430=      INDEX=INDEX+1
1440=      VAR(NVAR)=VAR(NVAR)+VINCRE
1450=      GO TO 6
1460= 5     CALL EXIT
1470= 41    WRITE(6,40)
1480= 40    FORMAT(1X,/,1X,"THE DISK OF RADIUS A LIES IN THE X-Y PLANE
",/,
1490=      11X,"CENTERED AT THE ORIGIN.  THE CONVENTIONAL (R, THETA",/,
,1X,
1500=      1"PHI) COORDINATE SYSTEM IS USED IN THE FAR FIELD.  ",/,1X,
1510=      1"THE PLANE OF INCIDENCE OF THE PLANE WAVE IS THE",/,1X,
1520=      1"X-Z (PHI=0) PLANE.  THE POLARIZATION ANGLE (POL)",/,1X,
1530=      1"OF EINC IS MEASURED FROM THE PLANE OF INCIDENCE ",/,1X,
1540=      1"IN THE PHI-DIRECTION.  I.E., POL=0 IS THE PARALLEL",/,1X,
1550=      1"(THETA) CASE AND POL=90 IS THE PERPENDICULAR (PHI)",/,1X,
1560=      1"CASE.",2X,"THE RESULT IS A SOLUTION OF THE RIGOROUS",/,
1570=      11X,"EIGENFUNCTION SCATTERING PROBLEM.")
1580=      GO TO 4
1590= 42    CONTINUE
1600=      GO TO 4
1610=      END
1620=C
1630=C      OBLATE SPHEROIDAL ANGULAR FUNCTION,S,OF ARGUMENT 0;
1640=C      ORDERM,N; WITH N-M EVEN;UP TO ORDER N=M+2+NNMAX-2.
1650=C      (EQUAL TO PMN(0))
1660=C
1670=      SUBROUTINE SMND(M,NNMAX)
1680=      COMPLEX F4
1690=      COMMON EIG(50),D(50,50),DNEG(50),R1(50),F4(50)
1700=      1,SD(50)
1710=      SD(1)=1
1720=      IF(M.EQ.0.) GO TO 1
1730=      DO 2 MM=1,M
1740= 2      SD(1)=(2+MM-1)*SD(1)
1750= 1      DO 3 NN=1,NNMAX

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1760=      N=2*(NN-1)+M
1770= 3      SD(NN+1)=- (N+M+1)*SD(NN)/(N-M+2)
1780=      RETURN
1790=      END
1800=C
1810=C      SIN AND COSINE FUNCTIONS OF PHI
1820=C
1830=      SUBROUTINE CSPHI(MM,PHI)
1840=      COMPLEX F4,PSI,W,YD,YETAD,U,X
1850=      COMMON EIG(50),D(50,50),DNEG(50),R1(50),F4(50)
1860=      1,SD(50),P(50),SETA(50),SETAD(50),PSI(50),W(50),YD(50)
1870=      1,YETAD(50),U(50),X(50),CMPHI(50),SMPHI(50)
1880=      M=MM-1
1890=      CMPHI(MM)=COS(M*PHI)
1900=      SMPHI(MM)=SIN(M*PHI)
1910=      RETURN
1920=      END
1930=C
1940=C      OBLATE SPHEROIDAL EIGENVALUES OF ARGUMENT C, ORDER M,N
1950=C      WITH N-M EVEN UP TO ORDER N=M+2*NNMAX-2
1960=C
1970=      SUBROUTINE OBEIGN(C,M,NNMAX)
1980=      COMMON EIG(50)
1990=      DIMENSION IP(50),ALPHA(50),BETA(50),P(50)
2000= 4      CONTINUE
2010=      M2=2*M
2020=      C2=C*C
2030=      ACC=1.0E-05
2040=      NN2=NNMAX+2
2050=      NI=NN2+1
2060=      P(1)=1
2070=      IP(1)=1
2080=      DO 2 I00=1,NN2
2090=      IV=2*I00-1
2100=      IW=M2+2*I00
2110=      IX=M2+4*I00-1
2120=      ALPHA(I00)=(C2*(M2*(2*IV-1)+2*IV*(IV-1)-1))/(IX*(IX-4))
2130=      1-(M+IV-1)*(IV+M)
2140= 2      BETA(I00+1)=C2/IX*SQRT(IV*(IV+1)*IW*(IW-1)/(IX*(IX-4.0))
2150=      BETA(NN2+1)=0.
2160=      B0=ABS(ALPHA(1))+ABS(BETA(2))
2170=      DO 3 I00=2,NN2
2180=      A0=ABS(BETA(I00))+ABS(ALPHA(I00))+ABS(BETA(I00+1))
2190=      BETA(I00)=BETA(I00)+BETA(I00)
2200=      IF(A0.GT.B0) B0=A0
2210= 3      CONTINUE
2220=      A0=-B0
2230=      B0I=B0
2240= 13      CONTINUE
2250=      B0=B0I
2260=      DO 20 I00=1,NNMAX
2270=      N=2*I00-2+M
2280=      A=A0
2290=      B=B0
2300=      IERR=-1

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2310= 21   IIS=0
2320=      CD=(A+B)/2
2330=      IF (CD) 50,22,50
2340= 50   ERR=(P-A)/ABS(CD)
2350=      IERP=IERP+1
2360=      IF (IERP-60) 40,41,41
2370= 41   WRITE(6,42) N
2380= 42   FORMAT(2X,"ITERATIONS EXCEEDED FOR EIGENVALUE ",I3)
2390=      GO TO 700
2400= 40   IF (ERR-ACC) 24,24,22
2410= 22   P(2)=ALPHA(1)-CD
2420=      DO 5 I=3,NI
2430=      P(I)=(ALPHA(I-1)-CD-BETA(I-1)+(P(I-2)/P(I-1)))*P(I-1)
2440=      PMAG=ABS(P(I))
2450=      IF (PMAG.GT.1.0E+33) GO TO 7
2460= 5     CONTINUE
2470= 12    CONTINUE
2480=      DO 6 I=2,NI
2490=      IF (P(I)) 14,8,9
2500= 8     IF (P(I-1)) 9,9,14
2510= 14    IP(I)=-1
2520=      GO TO 10
2530= 9     IP(I)=1
2540= 10    IF (IP(I)-IP(I-1)) 6,11,6
2550= 11    IIS=IIS+1
2560= 6     CONTINUE
2570=      IF (IIS-I00) 16,15,15
2580= 15    A=CD
2590=      GO TO 21
2600= 16    B=CD
2610=      GO TO 21
2620= 24    BD=CD
2630= 700   EIG(I00)=-CD
2640= 20    CONTINUE
2650=      RETURN
2660= 7     NNMAX=I-4
2670=      NI=NNMAX+3
2680=      GO TO 4
2690=      END
2700=C
2710=C
2720=C
2730=C
2740=C
2750=      SUBROUTINE DBCOFN(C,M,MMMAX,NNMAX,IRRMX)
2760=      COMMON EIG(50),D(50,50)
2770=      DIMENSION DP(50)
2780=      C2=C+C
2790=      MM=M+1
2800=      DO 1 NN=1,NNMAX
2810=      N=M+2+NN-2
2820= 4      DP(IRRMX+3)=0
2830=      DP(IRRMX+2)=1.0E-30
2840=      D(NN,1)=0
2850=      D(NN,2)=1

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2860=      JJ=(N-M)/2+1
2870=      DO 107 LL=1,IRRMAX
2880=      L=LL-1
2890=      IF (LL.GE.JJ) L=IRRMAX+JJ-LL
2900=      IR=2*L
2910=      IRM=M+IR
2920=      AR=(M+IRM+2)*(M+IRM+1)*C2/((2*IRM+3)*(2*IRM+5))
2930=      BR=(2*IRM*(IRM+1)-2*M*M-1)*C2/((2*IRM-1)*
2940=      1 (2*IRM+3))-IRM*(IRM+1)
2950=      CR=IR*(IR-1)*C2/((2*IRM-3)*(2*IRM-1))
2960=      IF (LL-JJ) 105,106,106
2970= 105  D(NN,L+3)=- (CR*D(NN,L+1)+(BR+EIG(NN))*D(NN,L+2))/AR
2980=      DMAG=ABS(D(NN,L+3))
2990=      IF (DMAG.GT.1.0E+30) GO TO 3
3000=      GO TO 107
3010= 106  DP(L+1)=- (AR*DP(L+3)+(BR+EIG(NN))*DP(L+2))/CR
3020=      DMAG=ABS(DP(L+1))
3030=      IF (DMAG.GT.1.0E+30) GO TO 3
3040= 107  CONTINUE
3050=      DL=ABS(D(NN,JJ+1))
3060=      DL=ALOG10(DL)
3070=      DLP=ABS(DP(JJ+1))
3080=      DLP=ALOG10(DLP)
3090=      DL=ABS(DL)
3100=      DLP=ABS(DLP)
3110=      DL=DL+DLP
3120=      IF (DL.GT.30.) GO TO 5
3130=      CON=D(NN,JJ+1)/DP(JJ+1)
3140=      ACON=ABS(CON)
3150=      IF (ACON.GE.1.0E+32) GO TO 2
3160=      DO 118 J=JJ,IRRMAX
3170= 118  D(NN,J+2)=CON*DP(J+2)
3180=      F=1
3190=      IF (M) 198,198,199
3200= 199  DO 110 I=1,M
3210= 110  F=F*(M+I)
3220= 198  SUM=0
3230=      MMX=IRRMAX+1
3240=      DO 113 I=1,MMX
3250=      IR=2*I
3260=      SUM=SUM+F*D(NN,I+1)
3270=      IF (I-JJ) 113,197,113
3280= 197  FNM=F
3290= 113  F=(-F*(IR+2*M-1))/IR
3300=      ALF=FNM/SUM
3310=      DO 114 I=1,MMX
3320=      D(NN,I)=ALF*D(NN,I+1)
3330= 114  CONTINUE
3340= 1  CONTINUE
3350=      RETURN
3360= 3  IRRMAX=LL-1
3370=      GO TO 4
3380= 2  IRRMAX=IRRMAX-1
3390=      GO TO 4
3400= 5  NNMAX=NN-1

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3410=      RETURN
3420=      END
3430=C
3440=C      NEGATIVE D COEFFICIENT SUBROUTINE
3450=C
3460=      SUBROUTINE DNEG(C,M,NNMAX)
3470=      COMMON EIG(50),D(50,50),DNEG(50)
3480=      DO 4 NN=1,NNMAX
3490=      C2=C+C
3500=      IF (M.EQ.1) GO TO 2
3510=      DO 5 II=1,NNMAX
3520= 5      DNEG(II)=D(II,1)
3530=      GO TO 3
3540= 2      B1=1.0
3550=      B2=0.0
3560=      EI=EIG(NN)
3570=      DO 1 IRR=1,M
3580=      IR=2+IRR-2*M-2
3590=      AR=(2*M+IR+2)*(2*M+IR+1)+C2/((2*M+2+IR+3)
3600= 1*(2*M+2+IR+5))
3610=      BR=(M+IR)*(M+IR+1)-EI-(2*(M+IR)*(M+IR+1)-2*M*M-1)
3620= 1+C2/((2*M+2+IR-1)*(2*M+2+IR+3))
3630=      CR=(IR)*(IR-1)+C2/((2*M+2+IR-3)*(2*M+2+IR-1))
3640=      B3=B2
3650=      B2=B1
3660= 1      B1=(BR+B2-CR+B3)/AR
3670=      A=D(NN,1)/B1
3680=      DNEG(NN)=A
3690=      DDM=ABS(DNEG(NN))
3700=      IF (DDM.LT.1.0E-35) GO TO 6
3710= 4      CONTINUE
3720= 3      RETURN
3730= 6      NNMAX=NN-1
3740=      RETURN
3750=      END
3760=C
3770=C      OBLATE SPHEROIDAL RADIAL FUNCTION R(4) OF ARGUMENT C;
3780=C      ORDER M,N; WITH N-M EVEN; UP TO ORDER N=M+2+NNMAX-2.
3790=C      ALSO NORMALIZATION FUNCTION, N.
3800=C
3810=      SUBROUTINE DBRAD(C,M,10,MMMAX,NNMAX,IRRMX)
3820=      COMMON EIG(50),D(50,50),DNEG(50),R1(50),F4(50)
3830=      COMPLEX IX,R4,F4
3840=      IX=(0.0,1.0)
3850=      EEAC=1
3860=      EAC=1
3870=      EAC2=1
3880=      GRD=1
3890=      IF (M.EQ.0) GO TO 20
3900=      MAXM=M+1
3910=      DO 19 MM=2,MAXM
3920=      IM=MM-1
3930=      GRD=(2+IM-1)*(2+IM)+GRD
3940=      EEAC=(2+IM-1)*EEAC
3950=      EAC2=(2+IM-1)*(2+IM)+EAC2

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3960= 19   EAC=IM+EAC
3970=      IF (EEAC.GE.1.0E+17) GO TO 4
3980=      IF (EAC2.GE.1.0E+30) GO TO 4
3990= 20   DO 17 NN=1,NNMAX
4000=      N=2*(NN-1)+M
4010=      SUM=0
4020=      GR=GR0
4030=      ENORM=0.
4040=      DO 18 NR=1,IRMAX
4050=      IR=2*(NR-1)
4060=      SUMP=GR*D(NN,NR)
4070=      SUM=SUM+SUMP
4080=      DMAG=ABS(D(NN,NR))
4090=      IF (DMAG.LT.1.0E-30) GO TO 1
4100=      ENORMP=2*GR*(D(NN,NR))*2/(2*IR+2*M+1)
4110= 2     ENORM=ENORM+ENORMP
4120=      GRMAG=ABS(GR)
4130=      IF (GRMAG.GT.1.0E+30) GO TO 5
4140=      GR=(IR+2*M+1.)/(IR+1.)*(IR+2*M+2.)/(IR+2.)*GR
4150= 18   CONTINUE
4160=      R1(NN)=(-1)**(NN-1)*2**M*EAC+C**M*D(NN,1)/(2*M+1)*SUM
4170=      R2=(-1)**(NN-1)*(2*M-1)*EAC+C**(M-1)/(2*EAC2)*3.14159
4180=      1*(EEAC**2/SUM)*2**M/INEG(NN)
4190=      R2=ABS(R2)
4200=      IF (R2.GE.1.0E+30) GO TO 4
4210=      EEAC=(N+M+1)*EEAC/(N-M+2)
4220=      IF (EEAC.GE.1.0E+17) NNMAX=NN
4230=      R4=R1(NN)-IX*R2
4240=      A5=ABS(ENORM)
4250=      A1=ALOG10(A5)
4260=      A4=ABS(R2)
4270=      A2=ALOG10(A4)
4280=      A3=A1+A2
4290=      IF (A3.GT.30.) GO TO 3
4300=      A5=ABS(R1(NN))
4310=      A1=ALOG10(A5)
4320=      A3=ABS(A1)+ABS(A2)
4330=      IF (A3.GT.30.) GO TO 3
4340=      F4(NN)=1/(ENORM*R4)
4350= 17   CONTINUE
4360=      RETURN
4370= 1     ENORMP=0
4380=      GO TO 2
4390= 3     F4(NN)=(0.,0.)
4400=      GO TO 17
4410= 5     GR=0
4420=      GO TO 18
4430= 4     NNMAX=M
4440=      IQ=1
4450=      RETURN
4460=      END
4470=C
4480=C     PSI FUNCTION
4490=C
4500=      SUBROUTINE FPSI(M,NNMAX)

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4510=      COMPLEX IX,F4,PSI
4520=      COMMON EIG(50),D(50,50),DNEG(50),R1(50),F4(50)
4530=      1,SD(50),P(50),SETA(50),SETAD(50),PSI(50)
4540=      IX=(0.,1.)
4550=      MM=M+1
4560=      PSI(MM)=(0.,0.)
4570=      DO 1 NN=1,NNMAX
4580=      N=M+2+NN-2
4590=      PSI(MM)=PSI(MM)+IX+*(N)*SD(NN)+*2*F4(NN)
4600= 1     CONTINUE
4610=      RETURN
4620=      END
4630=C
4640=C      ASSOCIATED LEGENDRE POLYNOMIALS, OP, OF ARGUMENT ETAD;
4650=C      ORDERPM,N; WITH N-M EVEN; OF TO ORDER N=M+2+NNMAX-2.
4660=C
4670=C      SUBROUTINE POLYN(ETAD,M,NNMAX)
4680=C      DIMENSION PP(3)
4690=C      COMPLEX F4
4700=C      COMMON EIG(50),D(50,50),DNEG(50),R1(50),F4(50)
4710=C      1,SD(50),P(50)
4720=C      SQ=SQRT(1-ETAD*ETAD)
4730=C      PP(1)=0
4740=C      PP(2)=1
4750=C      IF(M.EQ.0) GO TO 1
4760=C      DO 2 L=1,M
4770=C 2     PP(2)=(2*L-1)*SQ*PP(2)
4780=C 1     P(1)=PP(2)
4790=C      DO 3 NN=2,NNMAX
4800=C      N=M+2+NN-3
4810=C      DO 4 L=1,2
4820=C      PP(3)=((2*N-1)*ETAD*PP(2)-(N+M-1)*PP(1))/(N-M)
4830=C      N=N+1
4840=C      PP(1)=PP(2)
4850=C 4     PP(2)=PP(3)
4860=C 3     P(NN)=PP(3)
4870=C      RETURN
4880=C      END
4890=C
4900=C      OBLATE SPHEROIDAL ANGULAR FUNCTIONS, S, OF ARGUMENTS
4910=C      C AND ETAD; ORDER M,N; WITH N-M EVEN; UP TO ORDER
4920=C      N=M+2+NNMAX-2.
4930=C
4940=C      SUBROUTINE DBANG(NNMAX,IRRMX)
4950=C      COMPLEX F4
4960=C      COMMON EIG(50),D(50,50),DNEG(50),R1(50),F4(50)
4970=C      1,SD(50),P(50),SETA(50),SETAD(50)
4980=C      DO 1 NN=1,NNMAX
4990=C      SETA(NN)=0
5000=C      DO 2 IRR=1,IRRMX
5010=C 2     SETA(NN)=SETA(NN)+D(NN,IRR)*P(IRR)
5020=C 1     CONTINUE
5030=C      RETURN
5040=C      END
5050=C

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5060=C      W FUNCTION
5070=C
5080=      SUBROUTINE FW(M,NNMAX)
5090=      COMPLEX IX,F4,W,PSI
5100=      COMMON EIG(50),D(50,50),DNEG(50),P1(50),F4(50)
5110=      1,SD(50),P(50),SETA(50),SETAD(50),PSI(50),W(50)
5120=      IX=(0.,1.)
5130=      MM=M+1
5140=      W(MM)=(0.,0.)
5150=      DO 1 NN=1,NNMAX
5160=      N=M+2+NN-2
5170=      1 W(MM)=W(MM)+IX+NN*SETAD(NN)+SD(NN)+F4(NN)
5180=      RETURN
5190=      END
5200=C
5210=C      Y FUNCTION
5220=C
5230=      SUBROUTINE FY(M,NNMAX)
5240=      COMPLEX F4,YD,YETAD,PSI,W
5250=      COMMON EIG(50),D(50,50),DNEG(50),R1(50),F4(50)
5260=      1,SD(50),P(50),SETA(50),SETAD(50),PSI(50),W(50),YD(50)
5270=      1,YETAD(50)
5280=      MM=M+1
5290=      YD(MM)=(0.,0.)
5300=      YETAD(MM)=(0.,0.)
5310=      DO 1 NN=1,NNMAX
5320=      N=M+2+NN-2
5330=      YD(MM)=YD(MM)+(-1)+NN*R1(NN)+SD(NN)+SETA(NN)+F4(NN)
5340=      1 YETAD(MM)=YETAD(MM)+(-1)+NN*R1(NN)+SETAD(NN)+SETA(NN)+F4(NN)
5350=      RETURN
5360=      END
5370=C
5380=C      S AND U FUNCTIONS
5390=C
5400=      SUBROUTINE FXU(MM,MMMAX)
5410=      COMPLEX IX,F4,PSI,W,YD,YETAD,U,X,PT
5420=      COMMON EIG(50),D(50,50),DNEG(50),R1(50),F4(50)
5430=      1,SD(50),P(50),SETA(50),SETAD(50),PSI(50),W(50),YD(50)
5440=      1,YETAD(50),U(50),X(50)
5450=      IX=(0.,1.)
5460=      IF(MM.EQ.1) GO TO 1
5470=      IF(MM.EQ.MMMAX) GO TO 2
5480=      PT=PSI(MM-1)+PSI(MM+1)
5490=      PTT=CABS(PT)
5500=      IF(PTT.EQ.0.) GO TO 3
5510=      U(MM)=2+IX+MM*(MM-2)+W(MM-1)+W(MM+1)/(PSI(MM-1)+PSI(MM+1))
5520=      X(MM)=2+IX+MM*(MM-2)+W(MM-1)-W(MM+1)/(PSI(MM-1)+PSI(MM+1))
5530=      RETURN
5540=      1 U(1)=-IX+W(2)/PSI(2)
5550=      X(1)=(0.,0.)
5560=      RETURN
5570=      2 U(MM)=IX+MM*(MM-2)+2+W(MM-1)/PSI(MM-1)
5580=      X(MM)=U(MM)
5590=      RETURN
5600=      3 MMAX=MM-1

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5610=      RETURN
5620=      END
5630=C
5640=C      Z FUNCTIONS
5650=C
5660=      SUBROUTINE FZ(MMMAX,Z,ZA,ZB,ZC,ZD)
5670=      COMPLEX IX,IQ,Z,ZA,ZB,ZC,ZD,F4,PSI,W,YD,YETAD,U,X
5680=      COMMON EIG(50),D(50,50),DNEG(50),R1(50),F4(50)
5690=      1,SQ(50),P(50),SETA(50),SETAD(50),PSI(50),W(50),YD(50)
5700=      1,YETAD(50),U(50),X(50),CMPHI(50),SMPHI(50)
5710=      IX=(0.,1.)
5720=      Z=(0.,0.)
5730=      ZA=Z
5740=      ZB=Z
5750=      ZC=Z
5760=      ZD=Z
5770=      MAXMM=MMMAX-1
5780=      DO 3 MM=1,MAXMM
5790=      M=MM-1
5800=      A=2
5810=      B=1
5820=      C=1
5830=      IF (MM.EQ.1) A=1
5840=      IF (MM.EQ.2) B=2
5850=      IF (MM.EQ.2) C=0
5860=      IQ=IX**(-M)
5870=      Z=Z+A*YETAD(MM)*CMPHI(MM)
5880=      IF (MM.EQ.1) GO TO 2
5890=      ZA=ZA+IQ*(U(MM+1)*CMPHI(MM+1)-B*U(MM-1)*CMPHI(MM-1))
5900=      1*YD(MM)
5910=      ZB=ZB+IQ*(U(MM+1)*SMPHI(MM+1)+U(MM-1)*SMPHI(MM-1))
5920=      1*YD(MM)
5930=      ZC=ZC+IQ*(X(MM+1)*SMPHI(MM+1)-X(MM-1)*SMPHI(MM-1))
5940=      1*YD(MM)
5950=      ZD=ZD+IQ*(X(MM+1)*CMPHI(MM+1)+C*X(MM-1)*CMPHI(MM-1))
5960=      1*YD(MM)
5970=      GO TO 1
5980= 2    ZA=ZA+U(2)*CMPHI(2)*YD(1)
5990=      ZB=ZB+U(2)*SMPHI(2)*YD(1)
6000=      ZC=ZC+X(2)*SMPHI(2)*YD(1)
6010=      ZD=ZD+X(2)*CMPHI(2)*YD(1)
6020= 1    CONTINUE
6030= 3    CONTINUE
6040=      RETURN
6050=      END

```

SCATTERING BY A METALLIC CIRCULAR DISK  
(HODGE -- VERSION 12/17/78)

(TYPE KA=0 TO STOP PROGRAM)  
(TYPE KA=-1 FOR A DESCRIPTION OF THE PARAMETERS)  
(NORMALIZATION:  $ESCAT=A \cdot EINC \cdot ENORM / (2 \cdot R) \cdot \exp(-J \cdot K \cdot R)$ )  
(ALL ANGLES IN DEGREES)

1. KA =1  
2. THETA INCIDENT =45  
3. POLARIZATION =0  
4. THETA SCATTERED =45  
5. PHI SCATTERED =0

WHICH VARIABLE IS TO BE INCREMENTED?1  
TYPE NUMBER OF CASES: 25  
WHAT IS THE INCREMENT?1

KA	CROSS SECTION		THETA		E NORM		PHI
	SIGMA/ (PI * A ** 2)	PHI	MAG	PHASE	MAG		
1.00	.351E+00	0.	.593E+00	-22.65	0.		
2.00	.903E+00	0.	.950E+00	-110.86	0.		
3.00	.263E+00	0.	.513E+00	148.85	0.		
4.00	.353E+00	0.	.594E+00	85.18	0.		
5.00	.334E+00	0.	.578E+00	-3.41	0.		
6.00	.447E+00	0.	.669E+00	-105.68	0.		
7.00	.149E+00	0.	.387E+00	148.80	0.		
8.00	.738E-01	0.	.272E+00	58.53	0.		
9.00	.659E-02	0.	.812E-01	43.29	0.		
10.00	.194E-01	0.	.139E+00	-45.88	0.		
11.00	.534E-01	0.	.231E+00	-95.93	0.		
12.00	.456E-01	0.	.213E+00	167.71	0.		
13.00	.102E+00	0.	.319E+00	87.48	0.		
14.00	.627E-01	0.	.250E+00	-11.15	0.		
15.00	.982E-01	0.	.313E+00	-103.23	0.		
16.00	.637E-01	0.	.252E+00	159.30	0.		
17.00	.570E-01	0.	.239E+00	86.09	0.		
18.00	.119E-01	0.	.109E+00	21.45	0.		
19.00	.800E-02	0.	.894E-01	-127.06	0.		
20.00	.244E-01	0.	.156E+00	124.65	0.		
21.00	.122E+00	0.	.349E+00	88.27	0.		
22.00	.172E+00	0.	.414E+00	71.90	0.		
23.00	.631E-01	0.	.251E+00	10.06	0.		
24.00	.116E+00	0.	.340E+00	-120.81	0.		
25.00	.164E+00	0.	.405E+00	72.44	0.		



AD-A080 367

AIR FORCE INST OF TECH WRIGHT-PATTERSON AFB OH SCH00--ETC F/6 17/9  
PREDICTED MICROWAVE ELECTRO-MAGNETIC BACKSCATTERING RETURNS FOR--ETC(U)  
DEC 79 F DIBARTOLOME0  
AFIT/GE/EE/79D-11

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NL

2 OF 2

AD  
A080 367



END

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3-80

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100=      PROGRAM SCATTER (INPUT,OUTPUT)
110=      COMPLEX A1,A2,B1,B2,H,P1,P2,F,U1,U2,X,Y,ESCAT,J
120=      REAL PI,THETA,A,B,D,E1,E2,F1,F2,G1,G2,C1,C2,KA,
130=      S1,S2,0,T1,T2,ARG,V,W,IER,MMBSJ0,MMBSJ1,MMBSJ2
140=      DIMENSION ESCATM(100),ANGLE(100),SIGMA(100)
150=      READ* THETA
160=      PI=3.141593
170=      THETA=(THETA/360.)*(2*PI)
180=      KA=0.
190=      A=1.414214
200=      B=-(PI/4)
210=      DO 20 I=1,25
220=      KA=KA+1.
230=      D=2*((2+KA)/PI)**(1./2.)
240=      E1=COS((PI/4.) - (THETA/2.))
250=      E2=COS((PI/4.) + (THETA/2.))
260=      F1=ABS(E1)
270=      F2=ABS(E2)
280=      G1=D*F1
290=      G2=D*F2
300=      CALL FRESNEL(G1,C1,S1)
310=      CALL FRESNEL(G2,C2,S2)
320=      H=CMPLX(0.,B)
330=      H=CEXP(H)
340=      P1=CMPLX(C1,S1)
350=      P2=CMPLX(C2,S2)
360=      A1=A*H*P1
370=      A2=A*H*P2
380=      ARG=2*KA*(SIN(THETA))
390=      V=(1 + SIN(THETA))/(2*SIN(THETA))
400=      W=(1 - SIN(THETA))/(2*SIN(THETA))
410=      MMBSJ2=(2/ARG)*MMBSJ1(ARG,IER) - MMBSJ0(ARG,IER)
420=      X=((A1**2)*W + (A2**2)*V)*MMBSJ1(ARG,IER)
430=      Y=((A1**2)*W - (A2**2)*V)*MMBSJ2
440=      J=(0.,1.)
450=      Y=Y*J
460=      Y1=REAL(Y)
470=      Y2=AIMAG(Y)
480=      X1=REAL(X)
490=      X2=AIMAG(X)
500=      ESCAT1=X1-Y1
510=      ESCAT2=X2-Y2
520=      ESCAT=CMPLX(ESCAT1,ESCAT2)
530=      ESCAT=ESCAT*J
540=      ESCAT=-ESCAT
550=      ESCATR=REAL(ESCAT)
560=      ESCATI=AIMAG(ESCAT)
570=      ESCATM(I)=((ESCATR**2.) + (ESCATI**2.))**(1./2.)
580=      SIGMA(I)= (ESCATM(I)**2.)
590=      ANGLE=ATAN(ESCATI/ESCATR)
600=      ANGLE(I)=((ANGLE)/(2*PI))*360.
610=      IF((ESCATR.LT.0.).AND.(ESCATI.GT.0.)) ANGLE(I)=ANGLE(I) + 1
80.      IF((ESCATR.LT.0.).AND.(ESCATI.LT.0.)) ANGLE(I)=ANGLE(I) - 1
80.
630= 20  CONTINUE
640=      KA=0.
650=      PRINT30

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```

660= 30   FORMAT(11X,/,/,8X,"KA",7X,"SIGMA/PI♦♦2",14X,"ESCAT",/,
670=      $37X,"MAG",9X,"PHASE",/,)
680=      DO 40 I=1,25
690=      KA=KA + 1.
700=      PRINT50,KA,SIGMA(I),ESCATM(I),ANGLE(I)
710= 50   FORMAT(" ",3X,F7.2,5X,E10.3,7X,E10.3,5X,F7.2)
720= 40   CONTINUE
730=      STOP
740=      END
750=      SUBROUTINE FRESNEL(AA,C,S)
760=      DIMENSION X(2000),Y(2000),ZX(2000),ZY(2000)
770=      INTEGER NDIM
780=      IF((AA.GT.-0.01).AND.(AA.LT.0.01))GO TO 6
790=      GO TO 11
800= 6     C=0.0
810=      S=0.0
820=      GO TO 4
830= 11    H=.01
840=      BB=ABS(AA)
850=      NDIM=BB/H + .1
860=      K=NDIM+1
870=      DO 10 I=1,K
880=      A=I*H - H
890=      X(I)=COS(1.57080♦(A♦♦2.))
900=      Y(I)=SIN(1.57080♦(A♦♦2.))
910= 10    CONTINUE
920=      SUM2=0.
930=      SUM4=0.
940= 1     HH=.5♦H
950=      C
960=      C
970=      INTEGRATION LOOP
980=      DO 2 I=1,NDIM
990=      SUM1=SUM2
1000=     SUM3=SUM4
1010=     SUM2=SUM2 + HH♦(X(I)+X(I+1))
1020=     SUM4=SUM4 + HH♦(Y(I)+Y(I+1))
1030= 2     ZX(I)=SUM1
1040= 3     ZY(I)=SUM3
1050=     ZX(NDIM)=SUM2
1060=     ZY(NDIM)=SUM4
1070= 7     IF(AA.GE.0.1)GO TO 8
1080=     C=(-1.)♦ZX(NDIM)
1090=     S=(-1.)♦ZY(NDIM)
1100= 8     GO TO 9
1110=     C=ZX(NDIM)
1120=     S=ZY(NDIM)
1130= 9     CONTINUE
1140= 4     RETURN
          END

```

KA	SIGMA/PI♦♦2	MAG	ESCAT PHASE
1.00	.409E+00	.640E+00	-.09
2.00	.104E+01	.102E+01	-92.45
3.00	.370E+00	.608E+00	2.59
4.00	.700E+00	.837E+00	105.75
5.00	.297E+00	.545E+00	-165.44
6.00	.265E+00	.514E+00	-67.00
7.00	.848E-01	.291E+00	14.66
8.00	.714E-01	.267E+00	100.69
9.00	.226E-01	.150E+00	147.33
10.00	.240E-01	.155E+00	-120.14
11.00	.444E-01	.211E+00	-80.24
12.00	.340E-01	.184E+00	12.08
13.00	.980E-01	.313E+00	90.71
14.00	.652E-01	.255E+00	-178.69
15.00	.110E+00	.332E+00	-85.81
16.00	.563E-01	.237E+00	1.94
17.00	.676E-01	.260E+00	94.77
18.00	.253E-01	.159E+00	168.41
19.00	.275E-01	.166E+00	-99.14
20.00	.169E-01	.130E+00	-51.91
21.00	.138E-01	.117E+00	45.27
22.00	.294E-01	.171E+00	97.22
23.00	.199E-01	.141E+00	-174.10
24.00	.513E-01	.226E+00	-91.91
25.00	.330E-01	.182E+00	-5.07

```

100=      PROGRAM DICK25(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
110=C      FAR FIELD SCATTERING BY A CIRCULAR METALLIC DISK
120=      DIMENSION LABEL(5),VAR(5)
130=      COMPLEX F4,PSI,W,YO,YETAO,U,X,Z,ZA,ZB,ZC,ZD
140=      1,IX,EPART,EPAPP,EPEPT,EPEPP,ESNT,ESNP
150=      COMMON EIG(50),D(50,50),DNEG(50),P1(50),F4(50)
160=      1,SD(50),P(50),SETA(50),SETAO(50),PSI(50),W(50),YO(50)
170=      1,YETAO(50),U(50),X(50),CMPHI(50),SMPHI(50)
180=      DATA LABEL(1)/10H   KA      /
190=      DATA LABEL(2)/10H THETA I  /
200=      DATA LABEL(3)/10H   POL    /
210=      DATA LABEL(4)/10H THETA S  /
220=      DATA LABEL(5)/10H   PHI S  /
230=      IX=(0,1)
240=      WRITE(6,11)
250= 11    FORMAT(1X,///,1X,"SCATTERING BY A METALLIC CIRCULAR DISK
260=      1/,5X,"(HODGE -- VERSION 12/17/78)")
270=      WRITE(6,26)
280= 26    FORMAT(1X,///,1X,"(TYPE KA=0 TO STOP PROGRAM)",/,
290=      11X,"(TYPE KA=-1 FOR A DESCRIPTION OF THE",
300=      1" PARAMETERS)",/,1X,"(NORMALIZATION: ESCAT=A+EINC+ENDOF
310=      1"/(2+R)+EXP(-J+K+R))",/,1X,"(ALL ANGLES IN DEGREES)")
320= 4      NDC=1
330=      INDEX=1
340=      WRITE(6,27)
350= 27    FORMAT(1X,///,1X,"1.  KA",14X,"= ")
360=      READ(5,*) VAR(1)
370=      IF(VAR(1).EQ.-1)GO TO 41
380=      IF(VAR(1).LE.0.)GO TO 5
390=      WRITE(6,28)
400= 28    FORMAT(1X,"2.  THETA INCIDENT  = ")
410=      READ(5,*) VAR(2)
420=      WRITE(6,29)
430= 29    FORMAT(1X,"3.  POLARIZATION    = ")
440=      READ(5,*) VAR(3)
450=      WRITE(6,30)
460= 30    FORMAT(1X,"4.  THETA SCATTERED = ")
470=      READ(5,*) VAR(4)
480=      WRITE(6,31)
490= 31    FORMAT(1X,"5.  PHI SCATTERED   = ")
500=      READ(5,*) VAR(5)
510=      WRITE(6,3)
520= 3      FORMAT(1X,///,1X,"WHICH VARIABLE IS TO BE INCREMENTED?")
530=      READ(5,*) NVAR
540=      IF((NVAR.LE.0).OR.(NVAR.GT.5))GO TO 23
550=      WRITE(6,21)
560= 21    FORMAT(1X,"TYPE NUMBER OF CASES:")
570=      READ(5,*) NDC
580=      WRITE(6,32)
590= 32    FORMAT(1X,"WHAT IS THE INCREMENT?")
600=      READ(5,*) VINCRE
610= 23    CONTINUE
620=      WRITE(6,24) LABEL(NVAR)
630= 24    FORMAT(1X,///,A10,4X,"CROSS SECTION",22X,"E NORM",/,
640=      113X,"SIGMA/(PI+A**2)",11X,"THETA",15X,"PHI",/,13X,
650=      1"THETA",7X,"PHI",8X,"MAG",5X,"PHASE",6X,"MAG",5X,

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660=      1"PHASE",/
670= 6      C=VAR(1)
680=      MMMAX=45
690=      NNMAX=45
700=      IRRMAX=45
710=      THED=VAR(2)*3.14159/180
720=      ETAD=COS(THED)
730=      THE=VAR(4)*3.14159/180
740=      ETA=COS(THE)
750=      PHI=VAR(5)*3.14159/180
760=      ALF=VAR(3)*3.14159/180
770=      CALF=COS(ALF)
780=      SALF=SIN(ALF)
790= 34      DO 10 MM=1,MMMAX
800=          M=MM-1
810=          IQ=0
820=          CALL SMNO(M,NNMAX)
830=          CALL DBEIGN(C,M,NNMAX)
840=          CALL DBCOFN(C,M,MMMAX,NNMAX,IRRMAX)
850=          CALL DNEGN(C,M,NNMAX)
860=          CALL DBRAD(C,M,IQ,MMMAX,NNMAX,IRRMAX)
870=          IF(IQ.EQ.1)GO TO 10
880=          CALL FPSI(M,NNMAX)
890=          CALL POLYN(ETAD,M,IRRMAX)
900=          CALL DBANG(NNMAX,IRRMAX)
910=          DO 7 I=1,NNMAX
920= 7          SETA(I)=SETA(I)
930=          CALL FW(M,NNMAX)
940=          CALL POLYN(ETA,M,IRRMAX)
950=          CALL DBANG(NNMAX,IRRMAX)
960=          CALL FY(M,NNMAX)
970= 10          CONTINUE
980=          DO 13 MM=1,MMMAX
990=          CALL FXU(MM,MMMAX)
1000=          IF(MMMAX.LT.MM)GO TO 13
1010=          CALL CSPHI(MM,PHI)
1020= 13          CONTINUE
1030=          CALL FZ(MMMAX,Z,ZA,ZB,ZC,ZD)
1040=          EPART=ETA*(-2+Z*CMPHI(2)+ZA)
1050=          EPARP=-2+Z*SMPHI(2)+ZB
1060=          EPERT=ETA*(2+Z*SMPHI(2)-ZC)
1070=          EPERP=-2+Z*CMPHI(2)+ZD
1080=          IF(ETAD.EQ.0.)GO TO 2
1090=          ESNT=2+IX*(CALF*EPART/ETAD+SALF*EPERT)/C
1100=          ESNP=2+IX*(CALF*EPARP/ETAD+SALF*EPERP)/C
1110=          GO TO 8
1120= 2          ESNT=2+IX*SALF*EPERT/C
1130=          ESNP=2+IX*SALF*EPERP/C
1140= 8          EMAGT=CABS(ESNT)
1150=          EMAGP=CABS(ESNP)
1160=          SIGTHE=EMAGT+EMAGT
1170=          SIGPHI=EMAGP+EMAGP
1180=          E1=REAL(ESNT)
1190=          E2=AIMAG(ESNT)
1200=          IF(E1.EQ.0.)GO TO 16

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1210=      ARG=E2/E1
1220=      EPHAT=180/3.14159*ATAN(ARG)
1230=      IF((E1.LT.0.).AND.(E2.GT.0.)) EPHAT=EPHAT+180
1240=      IF((E1.LT.0.).AND.(E2.LT.0.)) EPHAT=EPHAT-180
1250=      GO TO 17
1260= 16    EPHAT=90
1270=      IF(E2.LT.0.) EPHAT=-90
1280= 17    E1=REAL(ESNP)
1290=      E2=AIMAG(ESNP)
1300=      IF(E1.EQ.0.) GO TO 18
1310=      ARG=E2/E1
1320=      EPHAP=180/3.14159*ATAN(ARG)
1330=      IF((E1.LT.0.).AND.(E2.GT.0.)) EPHAP=EPHAP+180
1340=      IF((E1.LT.0.).AND.(E2.LT.0.)) EPHAP=EPHAP-180
1350=      GO TO 19
1360= 18    EPHAP=90
1370=      IF(E2.LT.0.) EPHAP=-90
1380= 19    IF(NVAR.EQ.0) NVAR=1
1390=      IF(VAR(1).GT.15) GO TO 1000
1400=      GO TO 2000
1410= 1000  CALL SCATTER(VAR,EMAGT,EPHAT,SIGTHE)
1420= 2000  CONTINUE
1430=      WRITE(6,25) VAR(NVAR),SIGTHE,SIGPHI,EMAGT,EPHAT,
1440=      1EMAGP,EPHAP
1450= 25    FORMAT(1X,F7.2,2(1X,E10.3),2(1X,E10.3,1X,F7.2))
1460= 37    IF(INDEX.EQ.NOC) GO TO 4
1470=      INDEX=INDEX+1
1480=      VAR(NVAR)=VAR(NVAR)+VINCRE
1490=      GO TO 6
1500= 5     CALL EXIT
1510= 41    WRITE(6,40)
1520= 40    FORMAT(1X,/,1X,"THE DISK OF RADIUS A LIES IN THE X-Y PLANE
",/,
1530=      11X,"CENTERED AT THE ORIGIN. THE CONVENTIONAL (R, THETA,",/
,1X,
1540=      1"PHI) COORDINATE SYSTEM IS USED IN THE FAR FIELD. ",/,1X,
1550=      1"THE PLANE OF INCIDENCE OF THE PLANE WAVE IS THE",/,1X,
1560=      1"X-Z (PHI=0) PLANE. THE POLARIZATION ANGLE (POL)",/,1X,
1570=      1"OF EINC IS MEASURED FROM THE PLANE OF INCIDENCE ",/,1X,
1580=      1"IN THE PHI-DIRECTION. I.E., POL=0 IS THE PARALLEL",/,1X,
1590=      1"(THETA) CASE AND POL=90 IS THE PERPENDICULAR (PHI)",/,1X,
1600=      1"CASE.",2X,"THE RESULT IS A SOLUTION OF THE RIGOROUS",/,
1610=      11X,"EIGENFUNCTION SCATTERING PROBLEM.")
1620=      GO TO 4
1630= 42    CONTINUE
1640=      GO TO 4
1650=      END
1660=C      DEBLATE SPHEROIDAL ANGULAR FUNCTION,S,OF ARGUMENT 0;
1670=C      ORDERM,N; WITH N-M EVEN;UP TO ORDER N=M+2*NNMAX-2.
1680=C      (EQUAL TO PMN(0))
1690=C
1700=C
1710=      SUBROUTINE SMND(M,NNMAX)
1720=      COMPLEX F4
1730=      COMMON EIG(50),D(50,50),DNEG(50),R1(50),F4(50)
1740=      1,SD(50)
1750=      SD(1)=1

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1760=      IF (M.EQ.0.) GO TO 1
1770=      DO 2 MM=1,M
1780= 2      SD(1)=(2*MM-1)*SD(1)
1790= 1      DO 3 NN=1,NNMAX
1800=      N=2*(NN-1)+M
1810= 3      SD(NN+1)=- (N+M+1)*SD(NN)/(N-M+2)
1820=      RETURN
1830=      END
1840=C
1850=C      SIN AND COSINE FUNCTIONS OF PHI
1860=C
1870=      SUBROUTINE CSPHI(MM,PHI)
1880=      COMPLEX F4,PSI,W,YD,YETAD,U,X
1890=      COMMON EIG(50),D(50,50),DNEG(50),R1(50),F4(50)
1900=      1,SD(50),P(50),SETA(50),SETAD(50),PSI(50),W(50),YD(50)
1910=      1,YETAD(50),U(50),X(50),CMPHI(50),SMPHI(50)
1920=      M=MM-1
1930=      CMPHI(MM)=COS(M*PHI)
1940=      SMPHI(MM)=SIN(M*PHI)
1950=      RETURN
1960=      END
1970=C
1980=C      OBLATE SPHEROIDAL EIGENVALUES OF ARGUMENT C, ORDER M,N
1990=C      WITH N-M EVEN UP TO ORDER N=M+2*NNMAX-2
2000=C
2010=      SUBROUTINE DBEIGN(C,M,NNMAX)
2020=      COMMON EIG(50)
2030=      DIMENSION IP(50),ALPHA(50),BETA(50),P(50)
2040= 4      CONTINUE
2050=      M2=2*M
2060=      C2=C+C
2070=      ACC=1.0E-05
2080=      NN2=NNMAX+2
2090=      NI=NN2+1
2100=      P(1)=1
2110=      IP(1)=1
2120=      DO 2 IQQ=1,NN2
2130=      IV=2*IQQ-1
2140=      IW=M2+2*IQQ
2150=      IX=M2+4*IQQ-1
2160=      ALPHA(IQQ)=(C2*(M2*(2*IV-1)+2*IV*(IV-1)-1))/(IX*(IX-4))
2170=      1-(M+IV-1)*(IV+M)
2180= 2      BETA(IQQ+1)=C2/IX*SQRT(IV*(IV+1)*IW*(IW-1)/(IX*(IX-4.0))
2190=      BETA(NN2+1)=0.
2200=      BO=ABS(ALPHA(1))+ABS(BETA(2))
2210=      DO 3 IQQ=2,NN2
2220=      AO=ABS(BETA(IQQ))+ABS(ALPHA(IQQ))+ABS(BETA(IQQ+1))
2230=      BETA(IQQ)=BETA(IQQ)+BETA(IQQ)
2240=      IF(AO.GT.BO) BO=AO
2250= 3      CONTINUE
2260=      AO=-BO
2270=      BOI=BO
2280= 13      CONTINUE
2290=      BO=BOI
2300=      DO 20 IQQ=1,NNMAX

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2310=      N=2*100-2+M
2320=      A=AO
2330=      B=BO
2340=      IERR=-1
2350= 21    IIS=0
2360=      CD=(A+B)/2
2370=      IF(CD)50,22,50
2380= 50    ERR=(B-A)/ABS(CD)
2390=      IERR=IERR+1
2400=      IF(IERR-60)40,41,41
2410= 41    WRITE(6,42) N
2420= 42    FORMAT(2X,"ITERATIONS EXCEEDED FOR EIGENVALUE ",I3)
2430=      GO TO 700
2440= 40    IF(ERR-ACC)24,24,22
2450= 22    P(2)=ALPHA(1)-CD
2460=      DO 5 I=3,NI
2470=      P(I)=(ALPHA(I-1)-CD-BETA(I-1)+(P(I-2)/P(I-1)))P(I-1)
2480=      PMAG=ABS(P(I))
2490=      IF(PMAG.GT.1.0E+33) GO TO 7
2500= 5     CONTINUE
2510= 12    CONTINUE
2520=      DO 6 I=2,NI
2530=      IF(P(I))14,8,9
2540= 8     IF(P(I-1))9,9,14
2550= 14    IP(I)=-1
2560=      GO TO 10
2570= 9     IP(I)=1
2580= 10    IF(IP(I)-IP(I-1))6,11,6
2590= 11    IIS=IIS+1
2600= 6     CONTINUE
2610=      IF(IIS-100)16,15,15
2620= 15    A=CD
2630=      GO TO 21
2640= 16    B=CD
2650=      GO TO 21
2660= 24    BO=CD
2670= 700   EIG(100)=-CD
2680= 20    CONTINUE
2690=      RETURN
2700= 7     NNMAX=I-4
2710=      NI=NNMAX+3
2720=      GO TO 4
2730=      END
2740=C
2750=C
2760=C
2770=C
2780=C
2790=      SUBROUTINE OBCOFN(C,M,MMMAX,NNMAX,IRMAX)
2800=      COMMON EIG(50),D(50,50)
2810=      DIMENSION DP(50)
2820=      C2=C+C
2830=      MM=M+1
2840=      DO 1 NN=1,NNMAX
2850=      N=M+2+NN-2

```

OBLATE SPHEROIDAL EIGENFUNCTION EXPANSION COEFFICIENTS  
OF ARGUMENT C; ORDER M,N,R; WITH N-M EVEN, UP TO ORDER  
N=M+2+NNMAX-2, R=2+IRMAX+2

```

2860= 4      IF (IRPMAX+3)=0
2870=      IF (IRPMAX+2)=1.0E-30
2880=      D(NN,1)=0
2890=      D(NN,2)=1
2900=      JJ=(N-M)/2+1
2910=      DO 107 LL=1,IRPMAX
2920=      L=LL-1
2930=      IF (LL.GE.JJ) L=IRPMAX+JJ-LL
2940=      IR=2*L
2950=      IRM=M+IR
2960=      AR=(M+IRM+2)*(M+IRM+1)*C2/((2*IRM+3)*(2*IRM+5))
2970=      BR=(2*IRM*(IRM+1)-2*M*M-1)*C2/((2*IRM-1)*
2980=      1 (2*IRM+3))-IRM*(IRM+1)
2990=      CR=IR*(IR-1)*C2/((2*IRM-3)*(2*IRM-1))
3000=      IF (LL-JJ) 105,106,106
3010= 105    D(NN,L+3)=-(CR*D(NN,L+1)+(BR+EIG(NN))*D(NN,L+2))/AR
3020=      DMAG=ABS(D(NN,L+3))
3030=      IF (DMAG.GT.1.0E+30) GO TO 3
3040=      GO TO 107
3050= 106    DP(L+1)=-(AR*DP(L+3)+(BR+EIG(NN))*DP(L+2))/CR
3060=      DMAG=ABS(DP(L+1))
3070=      IF (DMAG.GT.1.0E+30) GO TO 3
3080= 107    CONTINUE
3090=      DL=ABS(D(NN,JJ+1))
3100=      DL=ALOG10(DL)
3110=      DLF=ABS(DP(JJ+1))
3120=      DLF=ALOG10(DLF)
3130=      DL=ABS(DL)
3140=      DLF=ABS(DLF)
3150=      DL=DL+DLF
3160=      IF (DL.GT.30.) GO TO 5
3170=      CON=D(NN,JJ+1)/DP(JJ+1)
3180=      ACON=ABS(CON)
3190=      IF (ACON.GE.1.0E+32) GO TO 2
3200=      DO 118 J=JJ,IRPMAX
3210= 118    D(NN,J+2)=CON*DP(J+2)
3220=      F=1
3230=      IF (M) 198,198,199
3240= 199    DO 110 I=1,M
3250= 110    F=F*(M+I)
3260= 198    SUM=0
3270=      MMX=IRPMAX+1
3280=      DO 113 I=1,MMX
3290=      IR=2*I
3300=      SUM=SUM+F*D(NN,I+1)
3310=      IF (I-JJ) 113,197,113
3320= 197    FNM=F
3330= 113    F=(-F*(IR+2*M-1))/IR
3340=      ALF=FNM/SUM
3350=      DO 114 I=1,MMX
3360=      D(NN,I)=ALF*D(NN,I+1)
3370= 114    CONTINUE
3380= 1      CONTINUE
3390=      RETURN
3400= 3      IRRMAX=LL-1

```

```

3410=      GO TO 4
3420= 2    IPRMAX=IPRMAX-1
3430=      GO TO 4
3440= 5    NNMAX=NN-1
3450=      RETURN
3460=      END
3470=C
3480=C    NEGATIVE D COEFFICIENT SUBROUTINE
3490=C
3500=      SUBROUTINE DNEG(C,M,NNMAX)
3510=      COMMON EIG(50),D(50,50),DNEG(50)
3520=      DO 4 NN=1,NNMAX
3530=      C2=C+C
3540=      IF (M.GE.1) GO TO 2
3550=      DO 5 II=1,NNMAX
3560= 5     DNEG(II)=D(II,1)
3570=      GO TO 3
3580= 2     B1=1.0
3590=      B2=0.0
3600=      EI=EIG(NN)
3610=      DO 1 IRR=1,M
3620=      IR=2+IRR-2*M-2
3630=      AR=(2*M+IR+2)*(2*M+IR+1)*C2/((2*M+2*IR+3)
3640=      1*(2*M+2*IR+5))
3650=      BR=(M+IR)*(M+IR+1)-EI-(2*(M+IR)*(M+IR+1)-2*M*M-1)
3660=      1*C2/((2*M+2*IR-1)*(2*M+2*IR+3))
3670=      CR=(IR)*(IR-1)*C2/((2*M+2*IR-3)*(2*M+2*IR-1))
3680=      B3=B2
3690=      B2=B1
3700= 1     B1=(BR+B2-CR*B3)/AR
3710=      A=D(NN,1)/B1
3720=      DNEG(NN)=A
3730=      IDN=ABS(DNEG(NN))
3740=      IF (IDN.LT.1.0E-35) GO TO 6
3750= 4     CONTINUE
3760= 3     RETURN
3770= 6     NNMAX=NN-1
3780=      RETURN
3790=      END
3800=C
3810=C    OBLATE SPHEROIDAL RADIAL FUNCTION R(4) OF ARGUMENT C:
3820=C    ORDER M,N; WITH N-M EVEN; UP TO ORDER N=M+2*NNMAX-2.
3830=C    ALSO NORMALIZATION FUNCTION, N.
3840=C
3850=      SUBROUTINE DBRAD(C,M,IQ,MMMAX,NNMAX,IPRMAX)
3860=      COMMON EIG(50),D(50,50),DNEG(50),R1(50),F4(50)
3870=      COMPLEX IX,R4,F4
3880=      IX=(0.0,1.0)
3890=      EEAC=1
3900=      EAC=1
3910=      EAC2=1
3920=      GRD=1
3930=      IF (M.EQ.0) GO TO 20
3940=      MAXM=M+1
3950=      DO 19 MM=2,MAXM

```

```

3960=      IM=MM-1
3970=      GRD=(2+IM-1)*(2+IM)*GRD
3980=      EEAC=(2+IM-1)*EEAC
3990=      EAC2=(2+IM-1)*(2+IM)*EAC2
4000= 19    EAC=IM+EAC
4010=      IF (EAC.GE.1.0E+17) GO TO 4
4020=      IF (EAC2.GE.1.0E+30) GO TO 4
4030= 20    DO 17 NN=1,NNMAX
4040=      N=2*(NN-1)+M
4050=      SUM=0
4060=      GR=GRD
4070=      ENORM=0.
4080=      DO 18 NR=1,IRMAX
4090=      IR=2*(NR-1)
4100=      SUMP=GR*D(NN,NR)
4110=      SUM=SUM+SUMP
4120=      DMAG=ABS(D(NN,NR))
4130=      IF (DMAG.LT.1.0E-30) GO TO 1
4140=      ENORMP=2*GR*(D(NN,NR))*2/(2+IR+2*M+1)
4150= 2      ENORM=ENORM+ENORMP
4160=      GRMAG=ABS(GR)
4170=      IF (GRMAG.GT.1.0E+30) GO TO 5
4180=      GR=(IR+2*M+1.)/(IR+1.)*(IR+2*M+2.)/(IR+2.)*GR
4190= 18    CONTINUE
4200=      R1(NN)=(-1)*(NN-1)*2*M+EAC*D(NN,1)/((2*M+1)*SUM)
4210=      R2=(-1)*(NN-1)*(2*M-1)*EAC*D(NN,M-1)/(2+EAC2)*3.14159
4220= 1      1*(EEAC*2/SUM)*2*M/DNEG(NN)
4230=      RR2=ABS(R2)
4240=      IF (RR2.GE.1.0E+30) GO TO 4
4250=      EEAC=(N+M+1)*EEAC/(N-M+2)
4260=      IF (EEAC.GE.1.0E+17) NNMAX=NN
4270=      R4=R1(NN)-IX*R2
4280=      A5=ABS(ENORM)
4290=      A1=ALOG10(A5)
4300=      A4=ABS(R2)
4310=      A2=ALOG10(A4)
4320=      A3=A1+A2
4330=      IF (A3.GT.30.) GO TO 3
4340=      A5=ABS(R1(NN))
4350=      A1=ALOG10(A5)
4360=      A3=ABS(A1)+ABS(A2)
4370=      IF (A3.GT.30.) GO TO 3
4380=      F4(NN)=1/(ENORM*R4)
4390= 17    CONTINUE
4400=      RETURN
4410= 1      ENORMP=0
4420=      GO TO 2
4430= 3      F4(NN)=(0.,0.)
4440=      GO TO 17
4450= 5      GR=0
4460=      GO TO 18
4470= 4      MMAX=M
4480=      IQ=1
4490=      RETURN
4500=      END

```

```

4510=C
4520=C      PSI FUNCTION
4530=C
4540=      SUBROUTINE FPSI(M,NNMAX)
4550=      COMPLEX IX,F4,PSI
4560=      COMMON EIG(50),D(50,50),DNEG(50),R1(50),F4(50)
4570=      1,SD(50),P(50),SETA(50),SETAD(50),PSI(50)
4580=      IX=(0.,1.)
4590=      MM=M+1
4600=      PSI(MM)=(0.,0.)
4610=      DO 1 NN=1,NNMAX
4620=      N=M+2*NN-2
4630=      PSI(MM)=PSI(MM)+IX** (N) *SD(NN)**2*F4(NN)
4640= 1      CONTINUE
4650=      RETURN
4660=      END
4670=C
4680=C      ASSOCIATED LEGENDRE POLYNOMIALS, OP, OF ARGUMENT ETAD;
4690=C      ORDERPM,N; WITH N-M EVEN; UP TO ORDER N=M+2*NNMAX-2.
4700=C
4710=      SUBROUTINE POLYN(ETAD,M,NNMAX)
4720=      DIMENSION PP(3)
4730=      COMPLEX F4
4740=      COMMON EIG(50),D(50,50),DNEG(50),R1(50),F4(50)
4750=      1,SD(50),P(50)
4760=      SQ=SQRT(1-ETAD*ETAD)
4770=      PP(1)=0
4780=      PP(2)=1
4790=      IF(M.EQ.0) GO TO 1
4800=      DO 2 L=1,M
4810= 2      PP(2)=(2*L-1)*SQ*PP(2)
4820= 1      P(1)=PP(2)
4830=      DO 3 NN=2,NNMAX
4840=      N=M+2*NN-3
4850=      DO 4 L=1,2
4860=      PP(3)=((2*N-1)*ETAD*PP(2)-(N+M-1)*PP(1))/(N-M)
4870=      N=N+1
4880=      PP(1)=PP(2)
4890= 4      PP(2)=PP(3)
4900= 3      P(NN)=PP(3)
4910=      RETURN
4920=      END
4930=C
4940=C      OBLATE SPHEROIDAL ANGULAR FUNCTIONS, S, OF ARGUMENTS
4950=C      C AND ETAD; ORDER M,N; WITH N-M EVEN; UP TO ORDER
4960=C      N=M+2*NNMAX-2.
4970=C
4980=      SUBROUTINE OBANG(NNMAX,IRPMAX)
4990=      COMPLEX F4
5000=      COMMON EIG(50),D(50,50),DNEG(50),R1(50),F4(50)
5010=      1,SD(50),P(50),SETA(50),SETAD(50)
5020=      DO 1 NN=1,NNMAX
5030=      SETA(NN)=0
5040=      DO 2 IRP=1,IRPMAX
5050= 2      SETA(NN)=SETA(NN)+D(NN,IRP)*P(IRP)

```

```

5060= 1    CONTINUE
5070=      RETURN
5080=      END
5090=C
5100=C      W FUNCTION
5110=C
5120=      SUBROUTINE FW(M,NNMAX)
5130=      COMPLEX IX,F4,W,PSI
5140=      COMMON EIG(50),D(50,50),DNEG(50),R1(50),F4(50)
5150= 1,SD(50),P(50),SETA(50),SETAD(50),PSI(50),W(50)
5160=      IX=(0.,1.)
5170=      MM=M+1
5180=      W(MM)=(0.,0.)
5190=      DO 1 NN=1,NNMAX
5200=      N=M+2+NN-2
5210= 1    W(MM)=W(MM)+IX+*N*SETAD(NN)+SD(NN)+F4(NN)
5220=      RETURN
5230=      END
5240=C
5250=C      Y FUNCTION
5260=C
5270=      SUBROUTINE FY(M,NNMAX)
5280=      COMPLEX F4,YO,YETAD,PSI,W
5290=      COMMON EIG(50),D(50,50),DNEG(50),R1(50),F4(50)
5300= 1,SD(50),P(50),SETA(50),SETAD(50),PSI(50),W(50),YO(50)
5310= 1,YETAD(50)
5320=      MM=M+1
5330=      YO(MM)=(0.,0.)
5340=      YETAD(MM)=(0.,0.)
5350=      DO 1 NN=1,NNMAX
5360=      N=M+2+NN-2
5370=      YO(MM)=YO(MM)+(-1)+*N*R1(NN)+SD(NN)+SETA(NN)+F4(NN)
5380= 1    YETAD(MM)=YETAD(MM)+(-1)+*N*R1(NN)+SETAD(NN)+SETA(NN)
5390=      RETURN
5400=      END
5410=C
5420=C      S AND U FUNCTIONS
5430=C
5440=      SUBROUTINE FXU(MM,MMMAX)
5450=      COMPLEX IX,F4,PSI,W,YO,YETAD,U,X,PT
5460=      COMMON EIG(50),D(50,50),DNEG(50),R1(50),F4(50)
5470= 1,SD(50),P(50),SETA(50),SETAD(50),PSI(50),W(50),YO(50)
5480= 1,YETAD(50),U(50),X(50)
5490=      IX=(0.,1.)
5500=      IF(MM.EQ.1) GO TO 1
5510=      IF(MM.EQ.MMMAX) GO TO 2
5520=      PT=PSI(MM-1)+PSI(MM+1)
5530=      PTT=CABS(PT)
5540=      IF(PTT.EQ.0.) GO TO 3
5550=      U(MM)=2+IX+*(MM-2)+(W(MM-1)+W(MM+1))/(PSI(MM-1)+PSI(MM+1))
5560=      X(MM)=2+IX+*(MM-2)+(W(MM-1)-W(MM+1))/(PSI(MM-1)+PSI(MM+1))
5570=      RETURN
5580= 1    U(1)=-IX+W(2)/PSI(2)
5590=      X(1)=(0.,0.)
5600=      RETURN

```

```

5610= 2      U(MM)=IX♦♦(MM-2)♦2♦W(MM-1)/PSI(MM-1)
5620=      X(MM)=U(MM)
5630=      RETURN
5640= 3      MMAX=MM-1
5650=      RETURN
5660=      END
5670=C
5680=C      Z FUNCTIONS
5690=C
5700=      SUBROUTINE FZ(MMAX,Z,ZA,ZB,ZC,ZD)
5710=      COMPLEX IX,IQ,Z,ZA,ZB,ZC,ZD,F4,PSI,W,YO,YETAD,U,X
5720=      COMMON EIG(50),D(50,50),DNEG(50),R1(50),F4(50)
5730=      1,SD(50),P(50),SETA(50),SETAD(50),PSI(50),W(50),YO(50)
5740=      1,YETAD(50),U(50),X(50),CMPHI(50),SMPHI(50)
5750=      IX=(0.,1.)
5760=      Z=(0.,0.)
5770=      ZA=Z
5780=      ZB=Z
5790=      ZC=Z
5800=      ZD=Z
5810=      MAXMM=MMAX-1
5820=      DO 3 MM=1,MAXMM
5830=      M=MM-1
5840=      A=2
5850=      B=1
5860=      C=1
5870=      IF(MM.EQ.1) A=1
5880=      IF(MM.EQ.2) B=2
5890=      IF(MM.EQ.2) C=0
5900=      IQ=IX♦♦(-M)
5910=      Z=Z+A♦YETAD(MM)♦CMPHI(MM)
5920=      IF(MM.EQ.1) GO TO 2
5930=      ZA=ZA+IQ♦(U(MM+1)♦CMPHI(MM+1)-B♦U(MM-1)♦CMPHI(MM-1))
5940=      1♦YO(MM)
5950=      ZB=ZB+IQ♦(U(MM+1)♦SMPHI(MM+1)+U(MM-1)♦SMPHI(MM-1))
5960=      1♦YO(MM)
5970=      ZC=ZC+IQ♦(X(MM+1)♦SMPHI(MM+1)-X(MM-1)♦SMPHI(MM-1))
5980=      1♦YO(MM)
5990=      ZD=ZD+IQ♦(X(MM+1)♦CMPHI(MM+1)+C♦X(MM-1)♦CMPHI(MM-1))
6000=      1♦YO(MM)
6010=      GO TO 1
6020= 2      ZA=ZA+U(2)♦CMPHI(2)♦YO(1)
6030=      ZB=ZB+U(2)♦SMPHI(2)♦YO(1)
6040=      ZC=ZC+X(2)♦SMPHI(2)♦YO(1)
6050=      ZD=ZD+X(2)♦CMPHI(2)♦YO(1)
6060= 1      CONTINUE
6070= 3      CONTINUE
6080=      RETURN
6090=      END
6100=      SUBROUTINE SCATTER(VAR,ESCATM,ANGLE,SIGMA)
6110=      COMPLEX A1,A2,B1,B2,H,P1,P2,R,U1,U2,X,Y,ESCAT,J
6120=      REAL PI,THETA,A,B,D,E1,E2,F1,F2,G1,G2,C1,C2,KA,
6130=      S$1,S2,0,T1,T2,ARG,V,W,IER,MMBSJ0,MMBSJ1,MMBSJ2
6140=      DIMENSION VAR(1)
6150=      PI=3.141593

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```

6160=      THETA=((VAR(2))/360.)*(2*PI)
6170=      KA=VAR(1)
6180=      A=1.414214
6190=      B=-(PI/4)
6200=      D=2+(((2*KA)/PI)**(1./2.))
6210=      E1=COS((PI/4.) - (THETA/2.))
6220=      E2=COS((PI/4.) + (THETA/2.))
6230=      F1=ABS(E1)
6240=      F2=ABS(E2)
6250=      G1=D+F1
6260=      G2=D+F2
6270=      CALL FRESNEL(G1,C1,S1)
6280=      CALL FRESNEL(G2,C2,S2)
6290=      H=CMPLX(0.,B)
6300=      H=CEXP(H)
6310=      P1=CMPLX(C1,S1)
6320=      P2=CMPLX(C2,S2)
6330=      A1=A+H*P1
6340=      A2=A+H*P2
6350=      ARG=2*KA*(SIN(THETA))
6360=      V=(1 + SIN(THETA))/(2*SIN(THETA))
6370=      W=(1 - SIN(THETA))/(2*SIN(THETA))
6380=      MMBSJ2=(2/ARG)*MMBSJ1(ARG,IER) - MMBSJ0(ARG,IER)
6390=      X=((A1**2)*W + (A2**2)*V)*MMBSJ1(ARG,IER)
6400=      Y=((A1**2)*W - (A2**2)*V)*MMBSJ2
6410=      J=(0.,1.)
6420=      Y=Y*J
6430=      Y1=REAL(Y)
6440=      Y2=AIMAG(Y)
6450=      X1=REAL(X)
6460=      X2=AIMAG(X)
6470=      ESCAT1=X1-Y1
6480=      ESCAT2=X2-Y2
6490=      ESCAT=CMPLX(ESCAT1,ESCAT2)
6500=      ESCAT=ESCAT*J
6510=      ESCAT=-ESCAT
6520=      ESCATR=REAL(ESCAT)
6530=      ESCATI=AIMAG(ESCAT)
6540=      ESCATM=((ESCATR**2.) + (ESCATI**2.))**(1./2.)
6550=      SIGMA=(ESCATM**2.)
6560=      ANGLE=ATAN(ESCATI/ESCATR)
6570=      ANGLE=((ANGLE)/(2*PI))*360.
6580=      IF((ESCATR.LT.0.).AND.(ESCATI.GT.0.)) ANGLE=ANGLE + 180.
6590=      IF((ESCATR.LT.0.).AND.(ESCATI.LT.0.)) ANGLE=ANGLE - 180.
6600=      RETURN
6610=      END
6620=      SUBROUTINE FRESNEL(AA,C,S)
6630=      DIMENSION X(2000),Y(2000),ZX(2000),ZY(2000)
6640=      INTEGER NDIM
6650=      IF((AA.GT.-0.01).AND.(AA.LT.0.01))GO TO 6
6660=      GO TO 11
6670= 6      C=0.0
6680=      S=0.0
6690=      GO TO 4
6700= 11      H=.01

```



```

6710=      BB=ABS(AA)
6720=      NDIM=BB/H + .1
6730=      K=NDIM+1
6740=      DO 10 I=1,K
6750=      A=I*H - H
6760=      X(I)=COS(1.57080*(A**2.))
6770=      Y(I)=SIN(1.57080*(A**2.))
6780= 10    CONTINUE
6790=      SUM2=0.
6800=      SUM4=0.
6810= 1     HH=.5*H
6820=C
6830=C      INTEGRATION LOOP
6840=      DO 2 I=1,NDIM
6850=      SUM1=SUM2
6860=      SUM3=SUM4
6870=      SUM2=SUM2 + HH*(X(I)+X(I+1))
6880=      SUM4=SUM4 + HH*(Y(I)+Y(I+1))
6890=      ZX(I)=SUM1
6900= 2     ZY(I)=SUM3
6910= 3     ZX(NDIM)=SUM2
6920=      ZY(NDIM)=SUM4
6930= 7     IF(AA.GE.0.1)GO TO 8
6940=      C=(-1.)*ZX(NDIM)
6950=      S=(-1.)*ZY(NDIM)
6960=      GO TO 9
6970= 8     C=ZX(NDIM)
6980=      S=ZY(NDIM)
6990= 9     CONTINUE
7000= 4     RETURN
7010=      END

```

SCATTERING BY A METALLIC CIRCULAR DISK  
(HODGE -- VERSION 12/17/78)

(TYPE KA=0 TO STOP PROGRAM)  
(TYPE KA=-1 FOR A DESCRIPTION OF THE PARAMETERS)  
(NORMALIZATION:  $ESCAT=A \cdot EINC \cdot ENORM / (2 \cdot R) \cdot \exp(-J \cdot K \cdot R)$ )  
(ALL ANGLES IN DEGREES)

1. KA = 440008 CM STORAGE USED  
2.168 CP SECONDS COMPILATION TIME1
2. THETA INCIDENT =45
3. POLARIZATION =0
4. THETA SCATTERED =45
5. PHI SCATTERED =0

WHICH VARIABLE IS TO BE INCREMENTED?1  
TYPE NUMBER OF CASES: 25  
WHAT IS THE INCREMENT?1

KA	CROSS SECTION SIGMA/(PI*A**2)		E NORM		PHI	
	THETA	PHI	MAG	PHASE	MAG	PHASE
1.00	.351E+00	0.	.593E+00	-22.65	0.	90.00
2.00	.903E+00	0.	.950E+00	-110.86	0.	90.00
3.00	.263E+00	0.	.513E+00	148.85	0.	90.00
4.00	.353E+00	0.	.594E+00	85.18	0.	90.00
5.00	.334E+00	0.	.578E+00	-3.41	0.	90.00
6.00	.447E+00	0.	.669E+00	-105.68	0.	90.00
7.00	.149E+00	0.	.387E+00	148.80	0.	90.00
8.00	.738E-01	0.	.272E+00	58.53	0.	90.00
9.00	.659E-02	0.	.812E-01	43.29	0.	90.00
10.00	.194E-01	0.	.139E+00	-45.88	0.	90.00
11.00	.534E-01	0.	.231E+00	-95.93	0.	90.00
12.00	.456E-01	0.	.213E+00	167.71	0.	90.00
13.00	.102E+00	0.	.319E+00	87.48	0.	90.00
14.00	.627E-01	0.	.250E+00	-11.15	0.	90.00
15.00	.982E-01	0.	.313E+00	-103.23	0.	90.00
16.00	.563E-01	0.	.237E+00	1.94	0.	90.00
17.00	.676E-01	0.	.260E+00	94.77	0.	90.00
18.00	.253E-01	0.	.159E+00	168.41	0.	90.00
19.00	.275E-01	0.	.166E+00	-99.14	0.	90.00
20.00	.169E-01	0.	.130E+00	-51.91	0.	90.00
21.00	.138E-01	0.	.117E+00	45.27	0.	90.00
22.00	.294E-01	0.	.171E+00	97.22	0.	90.00
23.00	.199E-01	0.	.141E+00	-174.10	0.	90.00
24.00	.513E-01	0.	.226E+00	-91.91	0.	90.00
25.00	.330E-01	0.	.182E+00	-5.07	0.	90.00

## Appendix C

### Program Listing for the Fast Fourier Transform (FFT)

The following is a program listing of the Fast Fourier Transform which performs the Direct and Inverse Discrete Fourier Transforms (Ref 13:331).

In the FFT subroutine argument list, X represents the complex array to be transformed, M represents the power of the two of which is the length of the FFT, and XI indicates whether a direct FFT or inverse FFT is to be performed. If XI is a plus one, the direct FFT is performed. If XI is minus one, the inverse FFT is performed.

An example of calling the FFT subroutine from a main routine is:

```
CALL FFT (DOG,10,1.)
```

In this Call statement, "DOG" is the complex array to be transformed, "10" is the power of two of which is the length of the FFT and "1." indicates that a Direct FFT will be performed.

A few precautions in using the FFT subroutine shown should be stated. Using the above example, the precautions are:

- (1) DOG must be a complex array with  $2^{10}$  complex locations.
- (2) The length of the FFT of DOG will have  $2^{10}$  complex locations.

- (3) The second parameter in the argument list, namely "10" must be a positive integer and must not have a decimal point.
- (4) The third and last parameter in the list, namely "1." must have a decimal point.
- (5) The length of the FFT must be confined to half the maximum number of real array locations that are possible with the computing machine used.

```

100=      SUBROUTINE FFT(X,M,XI)
110=      COMPLEX X(1),U,W,T
120=      N=2**M
130=      NV2=N/2
140=      NM1=N-1
150=      J=1
160=      DO 7 I=1,NM1
170=      IF(I.GE.J) GO TO 5
180=      T=X(J)
190=      X(J)=X(I)
200=      X(I)=T
210= 5      K=NV2
220= 6      IF(K.GE.J) GO TO 7
230=      J=J-K
240=      K=K/2
250=      GO TO 6
260= 7      J=J+K
270=      PI=3.14159265358979
280=      DO 20 L=1,M
290=      LE=2**L
300=      LE1=LE/2
310=      U=(1.0,0.0)
320=      W=CEXP(CMPLX(0.,-XI*PI/LE1))
330=      DO 20 J=1,LE1
340=      DO 10 I=J,N,LE
350=      IP=I+LE1
360=      T=X(IP)*U
370=      X(IP)=X(I)-T
380= 10      X(I)=X(I)+T
390= 20      U=U*W
400=      IF(XI.GT.0.) RETURN
410=      DO 30 I=1,N
420= 30      X(I)=X(I)/N
430=      RETURN
440=      END

```

## Appendix D

### Brief Review of the Selection of the FFT Parameters

The realization of the Discrete Fourier Transform on the modern digital computer in the form of the Fast Fourier Transform is a valuable tool in the field of linear systems. However, if the FFT parameters are not carefully picked according to certain rules, the results of performing an FFT on a sequence will provide erroneous results. So, it is wise to explore the relationships between FFT parameters in the time and frequency domains (Ref 15:282-284).

First of all, the parameters of interest are:

$T \triangleq$  increment between time samples (seconds).

$f_s \triangleq$  sampling rate (hertz) =  $1/T$

$F \triangleq$  increment between frequency samples (hertz) = frequency resolution

$t_p \triangleq$  record length (seconds) = effective period of time signal =  $1/F$

$f_0 \triangleq$  folding frequency =  $f_s/2$  (hertz)

$f_h \triangleq$  highest possible frequency in spectrum (hertz)

$N \triangleq$  number of samples in record

In order to avoid aliasing,

$$f_s \geq 2f_h \quad (D-1)$$

This implies that,

$$T \leq \frac{1}{2f_h} \quad (D-2)$$

For a desired frequency resolution

$$t_p = \frac{1}{F} \quad (D-3)$$

If  $f_h$  and  $F$  are both specified,  $N$  must satisfy

$$N \geq \frac{2f_h}{F} \quad (D-4)$$

The following is an example of using the FFT to obtain the DFT of the 10GHz pulse used in Chapter III and Chapter IV.

The pulse consists of five cycles of 10GHz R-f centered at the origin. The pulse is shown in Figure 11. Because of the periodic assumption of the time signal that the DFT makes, the negative half of the pulse is seen to be shifted up in time ending at  $t_p$ .

The first step is to estimate what the highest frequency component of the incident pulse is. From experiment it is obvious that the Fourier Transform of the pulse of R-F is a shifted  $\text{sinc}(x)$  frequency spectrum which never dies out as frequency increases. However, some criteria can be set where by if one of the minor lobes of the  $\text{sinc}(x)$  waveform goes below a certain percentage of the main lobe peak value, the frequency associated with that minor lobe would be considered the highest possible frequency in the spectrum of the

pulse. The continuous Fourier Transform of the 10GHz R-f pulse is

$$S(f) = 5 \times 10^{10} \frac{\sin [15.7 \times 10^{-10}(f-10^{10})]}{[15.7 \times 10^{-10}(f-10^{10})]} \quad (D-5)$$

At 13GHz up from the center frequency of 10GHz, or 23GHz, the magnitude of the sinc(x) function drops to 5 percent of its maximum value. This 23GHz will thus be taken as the highest possible frequency in the spectrum. The following relationships hold.

$$f_h = 23 \times 10^9 \text{ Hz} \quad (D-6)$$

$$f_s \geq 2f_h = 46 \times 10^9 \text{ Hz} \quad (D-7)$$

$$T \leq \frac{1}{f_s} = 0.0217 \text{ nsec}$$

If  $f_s$  is set at 100GHz

$$T = \frac{1}{f_s} = 0.01 \text{ nsec} \quad (D-9)$$

There will be ten samples per cycle of the incident pulse and thus 50 samples per full incident pulse. As was noted in Appendix C, to use the FFT subroutine shown there, the length



of the FFT must be a power of 2. If N is picked at 1024:

$$\begin{aligned} t_p &= 1024 \times T \\ &= 10.24 \text{ nsec} \end{aligned} \quad (D-10)$$

$$F = \frac{1}{t_p} = 97.656 \times 10^6 \text{ Hz} \quad (D-11)$$

Therefore the highest possible frequency in the spectrum of the pulse will occur at

$$\text{Sample \#} = \frac{f_h}{F} \approx 236 \quad (D-12)$$

Therefore, the frequency response of the scattering from the object the incident pulse is impinging on, can be cut off at sample 250 to sample  $1024 - 250 = 774$  without great loss in accuracy. Thus, when the DFT of the pulse and the sampled frequency response are multiplied and inverse transformed, the reflected pulse will be obtained. Because of the way the points were picked, the process of multiplication of transforms and inverse transforming will perform linear convolution and not cyclical convolution. For more information on cyclical convolution refer to the literature (Ref 15:p.284-295).

## VITA

Frank DiBartolomeo, Jr. was born on 5 December 1956 in New York City, New York, the son of Mr. and Mrs. Frank DiBartolomeo, Sr. He graduated from Bergen County Vocational-Technical High School, Hackensack, New Jersey in 1974. He received the degree of Bachelor of Science in Electrical Engineering and graduated Cum Laude from the New Jersey Institute of Technology, Newark, New Jersey in May 1978. Upon graduation, he received a commission in the United States Air Force through the ROTC program of which he was designated a Distinguished Graduate. He entered the School of Engineering, Air Force Institute of Technology, in June 1978. He received the degree of Master of Science in Electrical Engineering in December 1979. He is married to the former Kathy Currie and they have no children.

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1. REPORT NUMBER AFIT/GE/EE/79D-11	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) PREDICTED MICROWAVE ELECTRO-MAGNETIC BACKSCATTERING RETURNS FOR SIMPLE REFLECTIVE TARGETS		5. TYPE OF REPORT & PERIOD COVERED MS Thesis
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Frank DiBartolomeo, Jr. 2 Lt USAF		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Air Force Institute of Technology (AFIT/EN) Wright-Patterson AFB Ohio 45433		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Rome Air Development Center (OCTM) Griffiss AFB New York 13441		12. REPORT DATE Dec 1979
		13. NUMBER OF PAGES 121
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report)  Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES Approved for public release; IAW AFR 190-17  JOSEPH P. HIPPS, Maj, USAF Director of Public Affairs		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Circular Disk      Radar      Scattering Fast Fourier Transform      Radar Images      Signatures FFT      Radar Returns      Sphere Images      Radar Scattering      Synthetic Aperture Radar Linear Systems      Radar Signatures		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) The problem of identifying the perfectly conducting sphere and the perfectly conducting, infinitely thin circular disk via the backscattered far field from the object with a known incident signal is examined. A method utilizing synthetic aperture radar principles to obtain slant range versus cross range image plots of the discrete point scatterers on an object is discussed. A computer program to calculate the far field backscattering from the sphere is developed. Another computer program, found in the literature, which -		

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calculates the far field backscattering from the infinitely thin circular disk, is extended for  $ka \geq 15$ . In the above programs, the backscattering is obtained as a function of frequency and treated as the frequency response of a linear system. The input to this system is 5 cycles of 10 GHz R-F. The return signal is the output of the system and is obtained by using conventional linear system theory. Since a sphere is symmetric, the same return is obtained at all aspect angles. This return is examined. However, returns from the circular disk are different at different aspect angles. These returns from the disk are examined.

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